MA 428: Homework 1: Interpolation (Solutions)
Due: Friday, 18 September 2015

Theoretical

[*40 points*]
Section 5.1 – 8,9;

8. a Notice that

$$f(-3) = g(-3) = -23$$
$$f(1) = g(1) = -11$$
$$f(2) = g(2) = -23$$
$$f(5) = g(5) = 1$$

This shows that both $f(x)$ and $g(x)$ interpolate the data.

b This does not contradict the uniqueness of the interpolating polynomial because $f(x)$ and $g(x)$ are two different forms of the same polynomial. If you expand $g(x)$ you will obtain $f(x)$.

9 Let $P(x)$ denote the unique linear polynomial that interpolates f at $x = x_0$ and $x = x_1$. Then

$$\begin{aligned} |f(x) - P(x)| &= \left|\frac{f''(\xi)}{2!}\right| \cdot |(x - x_0)(x - x_1)| \quad \text{for some } \xi \in [x_0, x_1] \\ &\leq \frac{1}{2} \max_{x \in [x_0, x1]} |(x - x_0)(x - x_1)| \max_{\xi \in [x_0, x_1]} |f''(\xi)| \end{aligned}$$

The function $(x - x_0)(x - x_1) \leq 0$ for $x \in [x_0, x_1]$ since it is quadratic with roots at x_0, x_1 the maximum value of $|(x - x_0)(x - x_1)|$ achives its maximum at the vertex which occurs at $x = \frac{x_0 + x_1}{2}$ (this is an easy calcuation, take the derivative and set it to zero to find the vertex point). Thus,

$$\begin{aligned} \max_{x \in [x_0, x_1]} |(x - x_0)(x - x_1)| &= \left|\Big(\frac{x_0 + x_1}{2} - x_0\Big)\Big(\frac{x_0 + x_1}{2} - x_1\Big)\right| \\ &= \frac{1}{4}(x_1 - x_0)^2, \end{aligned}$$

and

$$|f(x) - P(x)| \leq \frac{1}{8} h^2 \max_{\xi \in [x_0, x_1]} |f''(\xi)|$$

where $h = x_1 - x_0$.

Section 5.3 – 3,8

3 Newton form: $P(x) = -1 + \frac{5}{2} + \frac{1}{2}(x - 2)(x - 4)$.

8 Newton form: $P(x) = \frac{2\sqrt{2}}{\pi} x + \frac{8 - 8\sqrt{2}}{\pi^2} x\big(x - \frac{\pi}{4}\big)$.
