MA 428: Homework 4: Composite Newton-Cotes Quadrature
Due: Wednesday, October 21

Computational

In this assignment you will numerically verify the convergence of the composite Trapezoidal rule and implement the composite Simpsons rule.

Please note that to numerically verify the rate of convergence of the method you need to give a table with at least 5 values showing the decay of the error as h decreases

Composite Trapezoidal Rule:

We divide the interval of integration $[a,b]$ into n subintervals with $h = \frac{(b-a)}{n}$ and $x_j = a + (j-1)h, 1 \leq j \leq n+1$. Applying the Trapezoidal rule on each subinterval, we obtain

$$\int_a^b f(x)\,dx = \frac{h}{2}\big[f(x_1) + 2\sum_{j=2}^{n} f(x_j) + f(x_{n+1})\big] - \frac{(b-a)}{12}h^2 f''(\xi)$$

Note that provided the integrand has two continuous derivatives, the composite trapezoidal rule has a convergence rate of $\mathcal{O}(h^2)$.

Composite Simpsons' Rule:

Since the basic Simpson's rule formula divides the interval $[a,b]$ into two pieces we must divide the interval $[a,b]$ into an even number of subintervals ($n = 2m$) to apply Simpsons rule in a composite manner m times, once over each subinterval $[x_{2j-1}, x_{2j+1}]$ for $j = 1, 2, 3, \cdots, m$. This results in the following composite Simposon's rule:

$$\int_a^b f(x)\,dx = \frac{h}{3}\big[f(x_1) + 4\sum_{j=1}^{m} f(x_{2j}) + 2\sum_{j=1}^{m-1} f(x_{2j+1}) + f(x_{2m+1})\big] - \frac{(b-a)h^4}{180} f^{(4)}(\xi).$$

I have provided a matlab function:

```
function approx_int = composite_quadrature(f,a,b,n,rule)
```

that takes as input the function f to be integrated on the interval $[a,b]$ with n subintervals using the Composite Trapezoidal rule (rule A) and the Composite Simpson's rule (rule B) that you will implement. Use the attached script `run_composite_rule.m` to generate a table of approximate integrals for increasing n, the errors and the ratio of the errors.

1. By making appropriate adjustments to the provided code, verify that the composite trapezoidal rule has a rate of convergence of $\mathcal{O}(h^2)$ (i.e second order) by approximating the integral

$$\int_0^1 \frac{4}{1+x^2}\,dx$$

2. Implement the composite Simpsons rule inside the provided `composite_quadrature` function.
3. What is the rate of convergence of the composite Simpsons' method?
4. Numerically verify the rate of convergence for the composite Simpsons rule.
5. Complete problem 29 in Section 6.5.

Theoretical

Section 6.5: 12a,12b

Submission

Email me your zipped m files, including your summary file with a discussion of your results for the computational part of the assignment. Your summary file must include all matlab output and answers to questions related to the output.