MA 428: Homework 5: Initial Value Problems

Due: Wednesday, November 4

Computational

1. Modify the provided code to apply Euler's method to approximate the solution of the given initial value problem over the indicated time interval using 8 time steps

$$\frac{dx}{dt} = \frac{-t \tan x}{1+t^2}, \quad x(0) = \pi/4, \quad ,0 \le t \le 1. \tag{1}$$

2. The true solution to (1) is:

$$x(t) = \arcsin \sqrt{(2+2t^2)^{-1}}.$$

   Create a table showing your approximate solution to (1), the true solution and the error at each time t_i. Comment on any trend you observe in the error as t increases.

3. Confirm that the global error associated with Euler's method is $\mathcal{O}(h)$ for the initial value problem (1).

4. Complete problem 27 in Section 7.2

Submission

Email me your zipped m files, including your summary file with a discussion of your results for the computational part of the assignment. Your summary file must include all matlab output and answers to questions related to the output.

Theoretical

1. Identify each of the following difference equations as representing a one-step or a multi-step method and as being implicit or explicit. In the case of the multi-step method identify the number of steps.

   (a) $\dfrac{w_{i+1} - w_i}{h} = \dfrac{3}{2} f(t_i, w_i) - \dfrac{1}{2} f(t_{i-1}, w_{i-1})$

   (b) $\dfrac{w_{i+1} - w_i}{h} = \dfrac{5}{12} f(t_{i+1}, w_{i+1}) + \dfrac{2}{3} f(t_i, w_i) - \dfrac{1}{12} f(t_{i-1}, w_{i-1})$

   (c) $\dfrac{w_{i+1} - w_{i-2}}{h} = \dfrac{3}{8} \big[f(t_{i+1}, w_{i+1}) + 3f(t_i, w_i) + f(t_{i-2}, w_{i-2}) \big]$
