Numerical differentiation

Case A : first order forward difference

- $$f'(x_0) = \frac{f(x_0 + h) - f(x_0)}{h} - \frac{h}{2} f''(\xi)$$

- Errors:

h	Error
1.0000e-01	7.0434e-02
1.0000e-02	6.4416e-03
1.0000e-03	6.3809e-04
1.0000e-04	6.3748e-05

- As h decreases by a factor of 10, the error decreases by a factor of approximately 10 aswell.
- This shows that the error in the approximation is first order in h as shown in the error term
- The error is proportional to h or as CS people say $\mathcal{O}(h)$.

Case B: Second order foward difference approximation

- $$f'(x_0) = \frac{-3f(x_0) + 4f(x_0+h) - f(x_0+2h)}{2h} + \frac{h^2}{3} f'''(\xi)$$

- Errors:

h	Error
1.0000e-01	1.3477e-02
1.0000e-02	1.3526e-04
1.0000e-03	1.3507e-06
1.0000e-04	1.3501e-08

- As h decreases by a factor of 10, the error decreases by a factor of 100.
- This error in the approximation is second order in h. $\mathcal{O}(h^2)$.

Case C: Second order central difference approximation

- $$f'(x_0) = \frac{f(x_0+h) - f(x_0-h)}{2h} + \frac{h^2}{6} f'''(\xi)$$

- Errors:

h	Error
1.0000e-01	6.7328e-03
1.0000e-02	6.7524e-05
1.0000e-03	6.7526e-07
1.0000e-04	6.7539e-09

- Second order approximation in h!
- Errors are smaller than the second order difference approximation. Why?