Case A : Composite Trapezoidal Rule

$$\int_a^b f(x)\,dx = \frac{h}{2}\left[f(x_1) + 2\sum_{j=2}^{n} f(x_j) + f(x_{n+1})\right] - \frac{(b-a)}{12}h^2 f''(\xi)$$

Matlab Implementation

We will compute $\int_0^{\pi} \sin x\,dx = 2$

1. We can define a function e.g
   ```
   function approx_int = composite_quadrature(f,a,b,n)
   ```
2. Break up the interval $[0, \pi]$ into n subintervals
   ```
   nodes = linspace(a,b,n+1);
   ```
3. Compute the approximate integral using the defined rule

   ```
   approx_int = (h/2)*(feval(f,nodes(1)) +
   2.0*sum(feval(f,nodes(2:n))) + feval(f,nodes(n+1)));
   ```

Error Analysis

- The error term is of the form $\frac{(b-a)}{12}h^2 f''(\xi)$

n	Approximate value	Error	Ratio of errors
2	1.570796326794897	0.429203673205103	
4	1.896118897937040	0.103881102062960	4.131681939078505
8	1.974231601945551	0.025768398054449	4.031337215586981
16	1.993570343772340	0.006429656227660	4.007741182739018
32	1.998393360970144	0.001606639029856	4.001929561139234
64	1.999598388640037	0.000401611359963	4.000482033192044
128	1.999899600184204	0.000100399815796	4.000120486053522
256	1.999974900235053	0.000025099764947	4.000030120146171

- This confirms that the order of convergence of the method is second order.