Math 151
FALL 2016
Exam 1
10/7/2016
Instructor: Dr. Prince Chidyagwai
Time Limit: 50 mins

Name (Print): SOLUTIONS

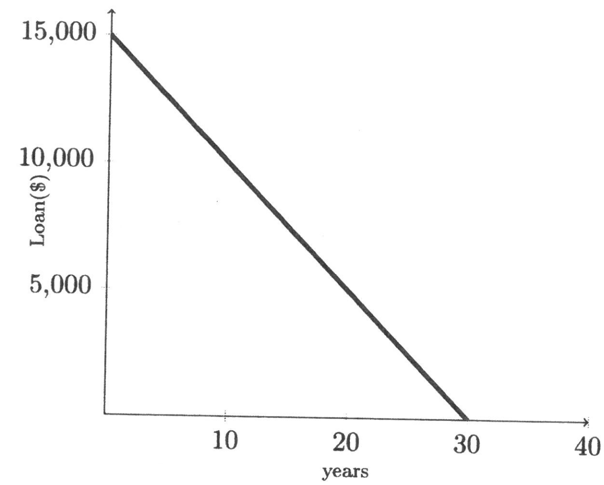
This exam contains 9 pages (including this cover page) and 7 problems including an OPTIONAL *bonus* problem worth 5 points. Check to see if any pages are missing. Enter all requested information on the top of this page.
You may *not* use your books, notes or a calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive any credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Problem	Points	Score
1	11	
2	12	
3	10	
4	22	
5	30	
6	15	
7	5	
Total:	105	

1. The graph shows the amount remaining to be paid on a 30 year loan over the life of the loan.



(a) (4 points) Find the slope of the graph and explain the meaning of the slope in the context of the loan.

$$m = \frac{\Delta \text{loan}}{\Delta t} = \frac{0 - 15{,}000}{30 - 0} = \frac{-\overset{500}{15{,}00\cancel{0}}}{3\cancel{0}} = -\$500/\text{year}$$

The loan payments are $500 per year

(b) (4 points) Identify the vertical and horizontal intercepts and explain their significance to the loan holder.

Vertical intercept = $15,000 is the amount borrowed

Horizontal intercept = 30years is the life span of the loan

(c) (3 points) Write a formula $R(t)$ for the amount of the loan remaining after t years.

$$R(t) = 15{,}000 - 500t$$

where t is in years

2. A used car is worth \$1,500 in 2012 and depreciates to \$1,000 in 2015

(a) (2 points) Find the *relative change* in the value of the car.

Relative change $= \dfrac{\Delta V}{V}$, where V is the value of the car

$$\frac{\$1000 - \$1{,}500}{1{,}500} = \frac{-500}{1{,}500} = -\frac{1}{3}$$

(b) (2 points) Find the *average rate of change* of the value of the car (give the appropriate units).

Average Rate of change $= \dfrac{\$1{,}000 - \$1{,}500}{2015 - 2012} = -\dfrac{\$500}{3\text{ yrs}}$

$= \dfrac{-500}{3}$ dollars per year

(c) (3 points) Assuming that the rate of depreciation of the car is always constant. Write down a formula $V(t)$ for the value of the car as a function of time t.

$$V(t) = 1{,}500 - \left(\frac{500}{3}\right)t$$

This means we assume that $V(t)$ is linear
constant rate of change

(d) (5 points) Suppose instead that the car loses its value at a 5% annual rate. What is the new formula for $V(t)$?

Constant % change $\Rightarrow$ exponential decay

negative due to loss of value

$$V(t) = 1{,}500\left(1 - 0.05\right)^t$$

$$= 1{,}500\left(0.95\right)^t$$

3. The revenue of Apple® went from \$25 billion in 2010 to \$75 billion in 2015.

(a) (5 points) What is the annual growth rate in revenue of Apple® over this period?

This implies we are looking for a constant % change. In class I did an example using train ridership (same idea).

we are looking for r.

Let $R(t)$ be revenue at time t since 2010

$R(t) = R_0 (a)^t$, where a is the growth factor, $a = 1+r$

This is always the initial amount

so $R(t) = 25(a)^t$ since $R(t) = 75$ in 2015

$\frac{75}{25} = \frac{\cancel{25}}{\cancel{25}} a^5 \Rightarrow a^5 = 3$ so $a = 3^{\frac{1}{5}}$, since $1+r = 3^{\frac{1}{5}}$

$r = 3^{\frac{1}{5}} - 1$

The growth rate is $3^{\frac{1}{5}} - 1$

(b) (5 points) What is the continuous growth rate per year of revenue?

continuous growth rate $\Rightarrow R(t) = R_0 e^{kt}$

we are looking for k

$R(t) = 25e^{kt}$

$\frac{75}{25} = \frac{25}{\cancel{25}} e^{k5}$ ← plug in 5

$3 = e^{5k}$

$\frac{1}{5}\ln(3) = k$

This is the continuous growth rate

Note: In class we said if we compare

$R_0 \underline{a}^t = R_0 \underline{e}^{kt}$

$a = e^k$

so $k = \ln(a)$ so

You could just take ln of your solution from 3(a)

4. In preparation for celebration of *Festivus*, Cosmo Kramer decides to get into the aluminum pole business. Kramer's factory has fixed costs of $6,000 with a marginal cost of $10.00 per pole. He then sales each aluminum pole $13.00.

   (a) (4 points) Let x be the number of poles manufactured and sold by Kramer. Write down the cost and revenue functions.

Let x be the number of poles sold.

$$C(x) = \underbrace{6,000}_{\text{fixed costs}} + \underbrace{10x}_{\text{Marginal cost}} \qquad R(x) = \underline{13x}$$

he sells each pole for $13.

   (b) (10 points) How many aluminum poles does Kramer need to sell to break even? Illustrate your answer on a plot of the cost and revenue functions.

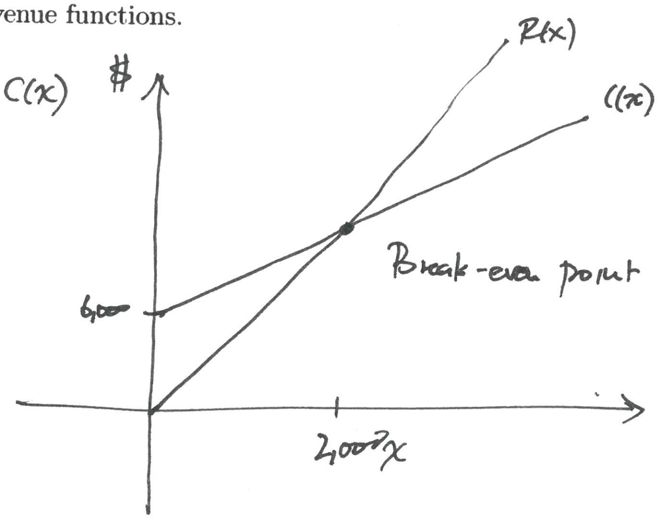
Break-even means $R(x) = C(x)$

$6,000 + 10x = 13x$

$\frac{6,000}{3} = \frac{3x}{3}$

$2,000 = x$

Kramer needs to sell 2,000 poles to break even.

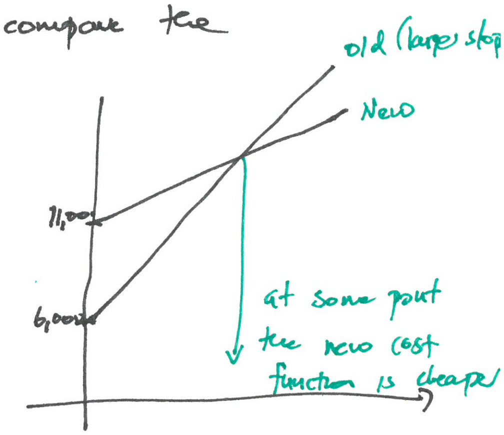


   (c) (8 points) Suppose a larger manufacturing factory offers Kramer the option of manufacturing with fixed costs of $11,000 but with a marginal cost lower marginal cost of $8 per pole. Assuming he continues to sell the poles at the same price, at what production level does it make financial sense to move to the larger plant?

Since the revenue is unchanged, we compare the cost functions

Old cost function $C(x) = 6,000 + 10x$

New cost function $C(x) = 11,000 + 8x$



$11,000 + 8x = 6,000 + 10x$

$5,000 = 2x$

$x = 2,500$

If Kramer is producing 2,500 poles it makes sense to move.

5. The demand and supply curves are given by

$$q = 100 - 3p, \text{ and } q = 2p - 50,$$

respectively.

(a) (5 points) Explain in economic terms the meaning of *equilibrium price and quantity.*

This is the point where quantity supplied is equal to quantity demanded by customers.

(b) (10 points) Find the equilibrium price and quantity.

Demand = Supply

$100 - 3p = 2p - 50$

$150 = 2p + 3p$

$150 = 5p$

$30 = p$

$q = 100 - 3p$

$= 100 - 3(30)$

$100 - 90$

$q = 10$

(c) (10 points) A tax of \$2.50 is imposed on the suppliers. Find the new equilibrium price and quantity.

The tax is on the suppliers, so this changes the supply equation.

suppliers get \$2.50 less

New supply is $2(\boxed{p - 2.50}) - 50 = 2p - 5 - 50$

$= 2p - 55$

Demand Equation Remains unchanged

$2p - 55 = 100 - 3p$

$5p = 155$

$p = \frac{\cancel{155}^{31}}{\cancel{5}}$

$p = 31$

(d) (5 points) How much of the \$2.50 tax is paid by the consumers?

The price goes up to 31, initialy it was \$30 so the consumer pay \$1.

6. (15 points) Suppose you deposit \$7,000 at a rate of 7% per year compounded continuously. Meanwhile, your wealthy neighbor invests \$14,000 in a bank that has a 4% rate compounded quarterly. When will the two accounts have the same balance?

You $7{,}000e^{0{,}07t}$ Neighbor $14{,}000\left(1+\frac{0{,}04}{4}\right)^{4t}$

We want to find t such that the 2 accounts have the same amounts so we set them equal.

$$\frac{7{,}000\, e^{0{,}07t}}{7{,}000} = \frac{14{,}000}{7{,}000}\left(1+0.01\right)^{4t}$$

$$e^{0{,}07t} = 2(1.01)^{4t}$$

$$\ln\left(e^{0.07t}\right) = \ln\left(2(1.01)^{4t}\right)$$

$$0{,}07t = \ln(2) + \ln(1.01)^{4t}$$

$$0{,}07t = \ln(2) + 4t\ln(1.01)$$ (grouping terms with t)

$$\left[0{,}07 - 4\ln(1.01)\right]t = \ln(2)$$

so

$$\boxed{t = \frac{\ln(2)}{0{,}07 - 4\ln(1.01)}}$$

7. (5 points) **(BONUS) Decide whether each of the following statements is TRUE or FALSE. Fully explain your reasoning, you may include a calculation or example if appropriate - you will receive NO credit for unsupported answers**

1. The Relative change is always positive

False relative change in P is $\frac{P_1 - P_0}{P_0}$

$P_1 - P_0$ may be negative

2. The cost function $C(q)$ is a decreasing function of quantity q.

False, $C(q)$ is always increasing, it always costs more to produce more goods.

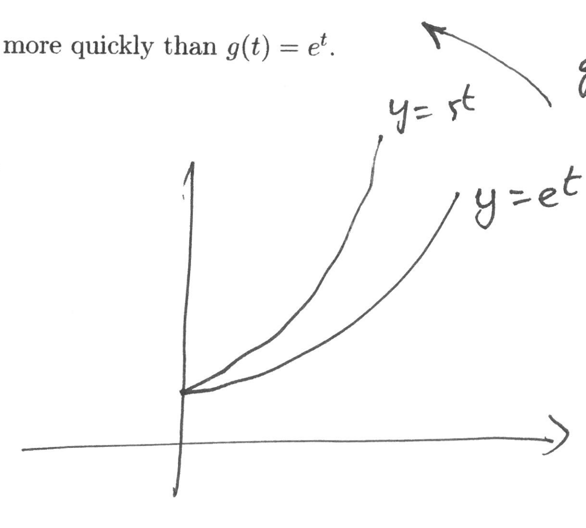
3. The doubling time of $P(t) = 3e^{5t}$ and $Q(t) = 6e^{5t}$ is the same.

True, in general the doubling time is dependent on the rate,

Indeed $\frac{2P_0}{P_0} = \frac{P_0 e^{kt}}{P_0} \Rightarrow t = \frac{\ln(2)}{k}$

4. The function $f(t) = 5^t$ grows more quickly than $g(t) = e^t$.

True $e < 5$



gets grows faster with increasing q.