Math 151
FALL 2016
Exam 2
11/9/2016
Instructor: Dr. Prince Chidyagwai
Time Limit: 50 mins

Name (Print): ____________________

This exam contains 7 pages (including this cover page) and 6 problems including an *optional* 5 point BONUS problem. Check to see if any pages are missing. Enter all requested information on the top of this page.
You may *not* use your books or notes on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive any credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Problem	Points	Score
1	20	
2	15	
3	25	
4	15	
5	25	
6	5	
Total:	105	

1. Suppose $V = f(t)$ represents the value of a stock in dollars t days from now.

   (a) (5 points) Interpret the following statements in terms of the price of the stock using appropriate units.

   (i) $f(4) = 50$

   The value of the stock is \$50 4 days from now

   (ii) $f'(4) = -0.5$.

   In 4 days, the value of the stock is decreasing at \$0.5 per day

   (b) (5 points) Use the statements in part(a) to calculate the relative rate of change of V at $t = 4$; interpret your result in terms of the value of the stock.

   $$\frac{f'(4)}{f(4)} = \frac{-0.5}{50} = \frac{-5}{500} = -0.01$$

   The stock is decreasing at a continuous rate of 1%.

   (c) (5 points) Use the statements in part (a) to estimate the value of the stock 6 days from now.

   $$f(6) \cong f(4) + f'(4)(6-4)$$
   $$50 + (-0.5)(2)$$
   $$50 + (-1.0) = \$49.$$

   (d) (5 points) Suppose on day 8, the value of the stock increases by \$5.00 in half a day of trading. A student wants to represent this information in the form

   $$f'(\underline{A}) = \underline{B}$$

   State the values of A and B. Give the appropriate units for each value.

   $$f'(x) = \frac{\text{Change in Value}}{\text{Change in time}} \qquad \frac{\$5.00}{0.5} = \$10/\text{day}$$

   $$f'(8) = \$10/\text{day}$$

2. After investing \$1000 at a annual rate of 7% compounded continuously in 2010, the balance on the account is given by $B = f(t)$ where t is the time measured since 2010.

   (a) (4 points) State the function $B = f(t)$ that gives the balance on the account in year t.

   $$B(t) = \$1000\, e^{0.07t}$$

   (b) (8 points) Find $\dfrac{dB}{dt}$ in 2015; interpret your answer in terms of the account balance.

   $$\frac{dB}{dt} = 0.07 \times 1000 e^{0.07t}$$

   $$= 70e^{0.07t}, \text{ @ } t=15 \quad \frac{dB}{dt} = 70e^{0.07 \times 15}.$$

   This is the rate of increase of the Balance of the account.

   (c) (3 points) **True** or **False**. The function $B'(t)$ is always positive. Explain.

   TRUE, the account is gaining interest.

3. Find the derivative of each of the following functions. If you apply a rule, leave your solution in the form of the rule. DO NOT simplify your solution.

   (a) (5 points) $y = \sqrt[3]{x} + 3x^2 + \dfrac{8}{x^3}$

   $$y = x^{\frac{1}{3}} + 3x^2 + 8x^{-3}$$

   $$\frac{dy}{dy} = \frac{1}{3}x^{-\frac{2}{3}} + 6x - 24x^{-4}$$

(b) (5 points) $y = 5(1.01)^x + e^{x^3}$

$$\frac{dy}{dx} = 5 \cdot \ln(1.01)\,(1.01)^x + 3x^2 e^{x^3}$$

(c) (5 points) $y = \dfrac{5x^2}{x^6 + 2^x}$

$$\frac{dy}{dx} = \frac{(x^6 + 2^x)\cdot(10x) - (5x^2)\left(6x^2 + \ln(2)\cdot 2^x\right)}{(x^6 + 2^x)^2}$$

(d) (5 points) $y = 5(3 - \sqrt{x}) \ln(x + 1)$

$$\frac{dy}{dx} = 5(3 - \sqrt{x})\,\frac{1}{x+1} + \ln(x+1)\left[5\left(0 - \tfrac{1}{2}x^{-1/2}\right)\right]$$

(e) (5 points) $y = \sqrt{(1 + 2e^{5x})}$

$$y = (1 + 2e^{5x})^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2}\left(1 + 2e^{5x}\right)^{-\frac{1}{2}}\left(0 + 5 \cdot 2e^{5x}\right)$$

4. A company's cost of producing q liters of a chemical is $C(q)$ dollars; this quantity can be sold for $R(q)$ dollars.

(a) (3 points) Suppose $C(2000) = 5000$ and $R(2000) = 7750$. What is the profit at a production level of 2000?

$$P(2000) = R(2000) - C(2000)$$
$$7750 - 5000$$
$$= \$2750 \ \cancel{\text{liters}}$$

(b) (7 points) If $C'(2000) = 2.1$ and $R'(2000) = 2.5$, what is the approximate change in profit if q is increased from 2000 to 2001?

$$P'(2000) = R'(2000) - C'(2000)$$
$$= 2.5 - 2.1$$
$$= \$0.4$$

(c) (5 points) Answer the following questions:

(i) Should the company increase or decrease production from $q = 2000$? Explain.

Increase because the profit increases by \$0.40

(ii) At what point should the company stop increasing or decreasing production?

When $R'(q) = C'(q)$ / $MR = MC$

5. The demand function and cost functions for a manufacturing plant are given by

$$p = 8e^{-0.003q}$$
$$C = 100 + 2q$$

Similar to WW problem #8

where q is the number of products sold at a price of p dollars per unit and C is the cost of producing q items.

(a) (5 points) Find the marginal cost at $q = 150$. Interpret this value in economic terms.

$MC(q) = 2$

It costs ~~$200~~ $2 to produce the 151 item.

(b) (5 points) Find the revenue function in terms of q.

Revenue = price x Quantity

$q \cdot 8e^{-0.003q}$

(c) (5 points) Find the rate of change of the revenue with respect to quantity sold at a quantity q; interpret the significance of this function in economic terms.

$R'(q) = q\left[8 \cdot (-0.003) e^{-0.003q}\right] + \left[8e^{-0.003q}\right]$

This is the rate of increase of revenue / Marginal Revenue.

(d) (5 points) Find the profit function in terms of q.

Profit = Revenue − Cost

$P(q) = q\,8e^{-0.003q} - (100 + 2q)$

(e) (5 points) Find the marginal profit function in terms of q. Interpret the significance of the function in economic terms.

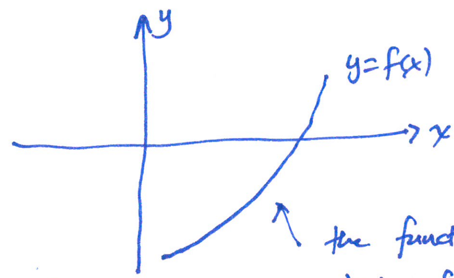
Marginal Profit $= P'(q)$

$P'(q) = R'(q) - C'(q)$ as found in (a) and (d)

6. (5 points) (BONUS) Determine whether each of the following statements is **True** or **False**. Fully explain your reasoning with a statement, example or calculation.

   (a) If a function $y = f(x)$ is negative on an interval, then the derivative is decreasing on that interval.

   False

   the function is negative but $f'(x) > 0$ and increasing, so $f''(x) > 0$

   (b) The marginal cost function is always greater than zero.

   TRUE, It always costs more to produce more items

   (c) If the revenue function is linear with a slope of 10, then the marginal revenue is zero.

   False, the Marginal Revenue is 10

   (d) If $A = f(B)$, then the units of $f'(B)$ are the units of B divided by the units of A.

   $f'(B) = \frac{dA}{dB}$ so units should be $\frac{\text{units of } A}{\text{units of } B}$

   (e) If the marginal cost is greater than the marginal revenue when the quantity produced is 1000 then increasing the quantity to 1001 will increase the profits.

   False, Marginal Revenue needs to be greater than Marginal cost