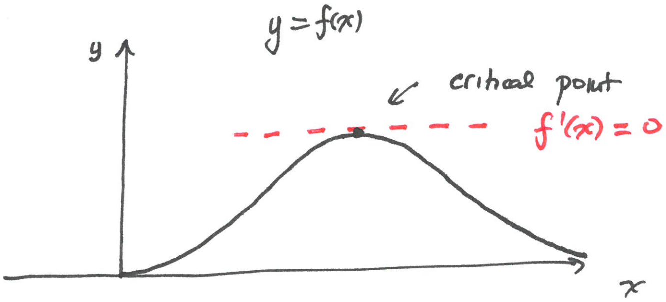
Homework #6 Solutions

4.1 1,-4, 5, 16, 17, 20, 23, 24-27, 38

1. $f'(x) = 0$ or $f'(x)$ is undefined at critical points



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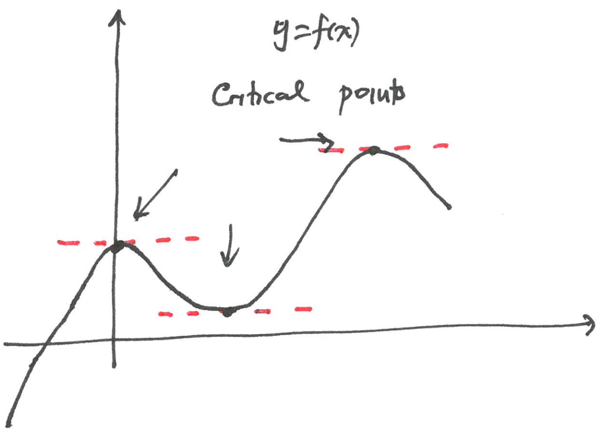

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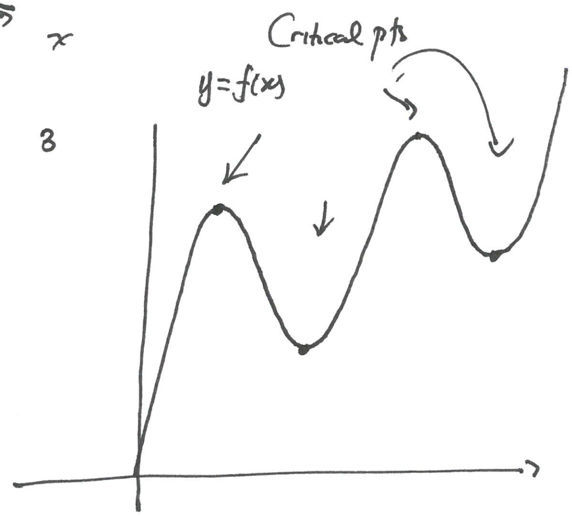

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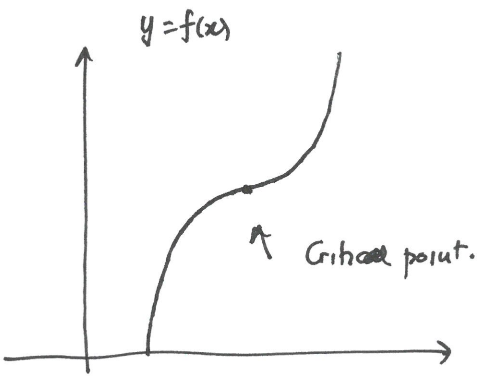
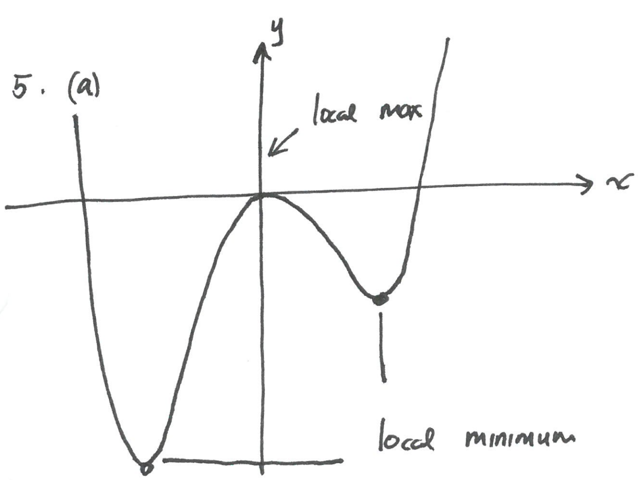
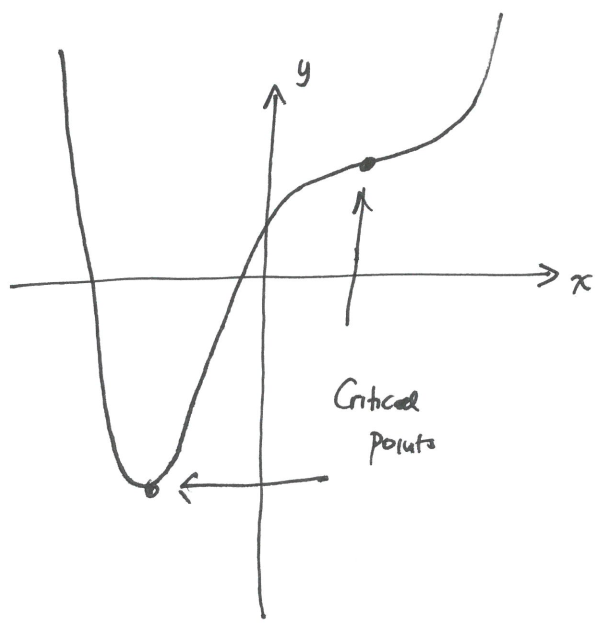


5. (a)



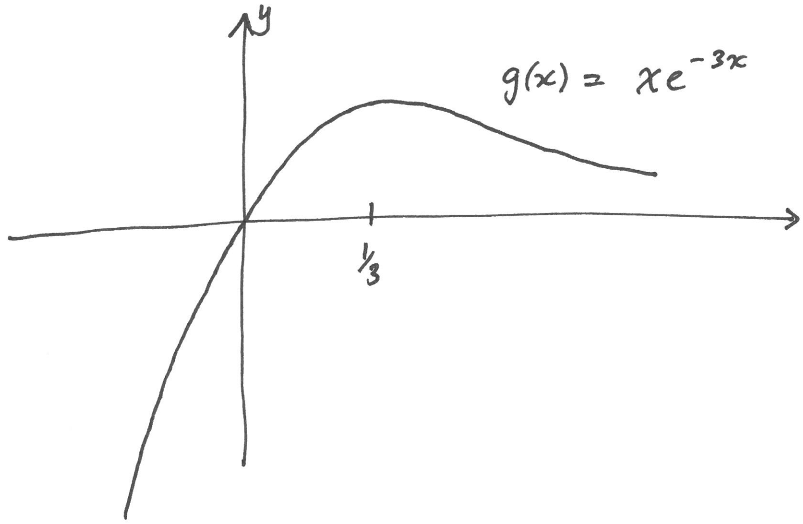


16. $g'(x) = e^{-3x} - 3x e^{-3x} = (1-3x)e^{-3x}$

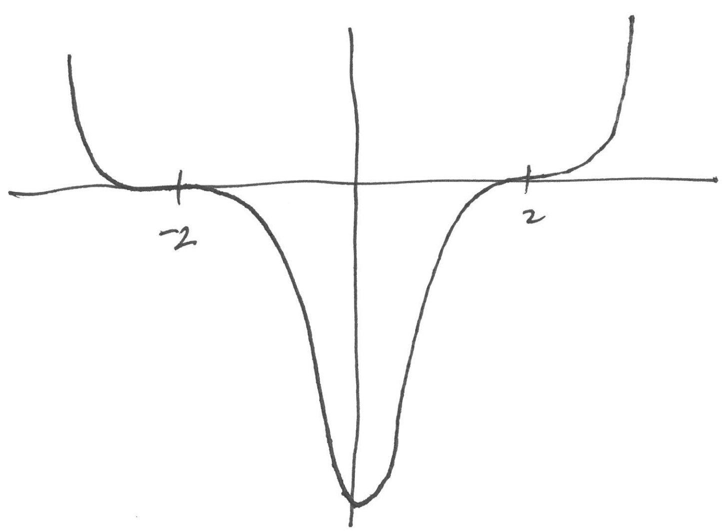
to find critical points set $g'(x) = 0$ and solve

since $e^{-3x} \neq 0$, $(1-3x)e^{-3x} = 0$ when $1 - 3x = 0$ so $x = \frac{1}{3}$

Since $g'(x) > 0$ to the left of $x = \frac{1}{3}$ and $g'(x) < 0$ to the right $g(\frac{1}{3})$ is a local max, infact



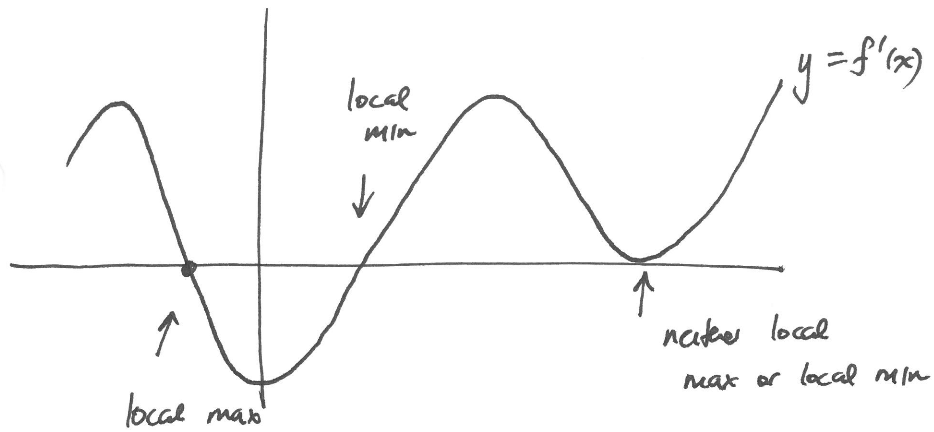
17. Your graph should look like this



Critical points are $x=0$, $x = \pm 2$

20. The graph is concave down at $x=1$, so $x=1$ is a local minimum.

23.



24. (a) Increasing for $x > 0$, decreasing for $x < 0$

(b) $f(0)$ is a local and global minimum, f has no global maximum.

25. (a) Increasing for all x

(b) No maxima or minima

26 (a) Decreasing for $x < 0$, increasing for $0 < x < 4$, decreasing for $x > 4$

(b) $f(0)$ is a local minimum, $f(4)$ is a local maximum.

27 (a) Decreasing for $x < -1$, increasing for $-1 < x < 0$

decreasing for $0 < x < 1$, increasing for $x > 1$

(b) $f(-1)$ and $f(1)$ are local minima, $f(0)$ is a local max.

36. Using product rule on $f(x) = axe^{bx}$, we have
$$f'(x) = ae^{bx} + abxe^{bx}$$
$$= ae^{bx}(1 + bx)$$

We want $f(\frac{1}{3}) = 1$ and $f'(\frac{1}{3}) = 0$ (because it has to be a max)
$$f(\tfrac{1}{3}) = a\left(\tfrac{1}{3}\right)e^{b/3} = 1$$
$$f'(\tfrac{1}{3}) = ae^{b/3}\left(1 + b/3\right) = 0$$

Since $ae^{b/3} \neq 0$

$$\frac{ae^{b/3}(1 + b/3)}{ae^{b/3}} = \frac{0}{ae^{b/3}}$$

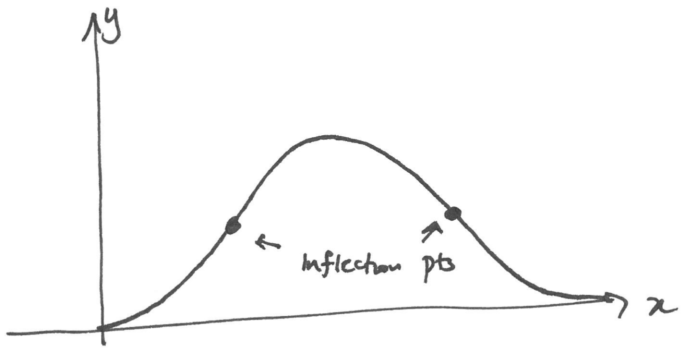
$$1 + b/3 = 0 \Rightarrow b = -3$$

Plug $b=-3$ into $a\left(\frac{1}{3}\right)e^{b/3} = 1$ gives $a = 3e$.

so $f(x) = \cancel{3e^x}\ 3ex\,e^{-3x}$.

4.2 1–4, 12, 13, 15, 24

1. Inflection points are places where f changes concavity



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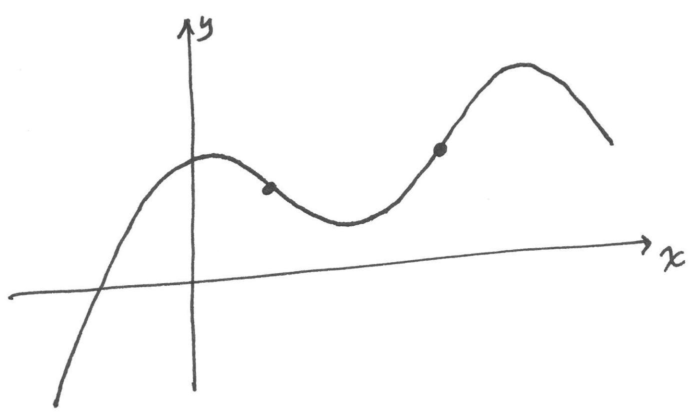

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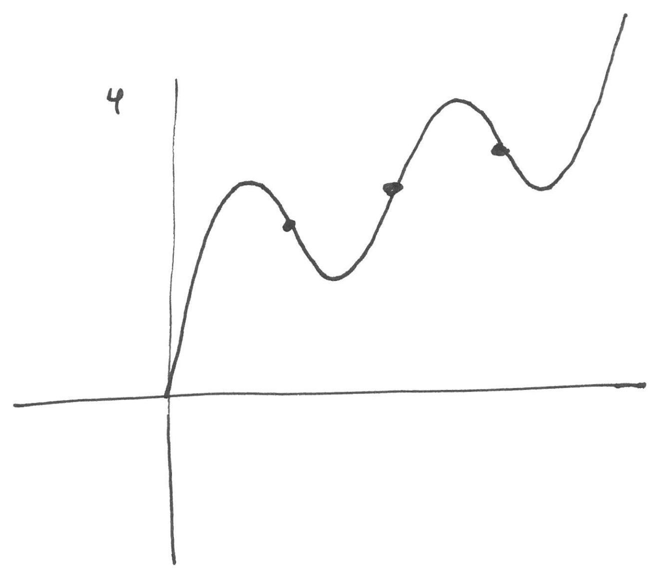


4.2

12. Critical point at $x = \frac{5}{2}$ (local min)

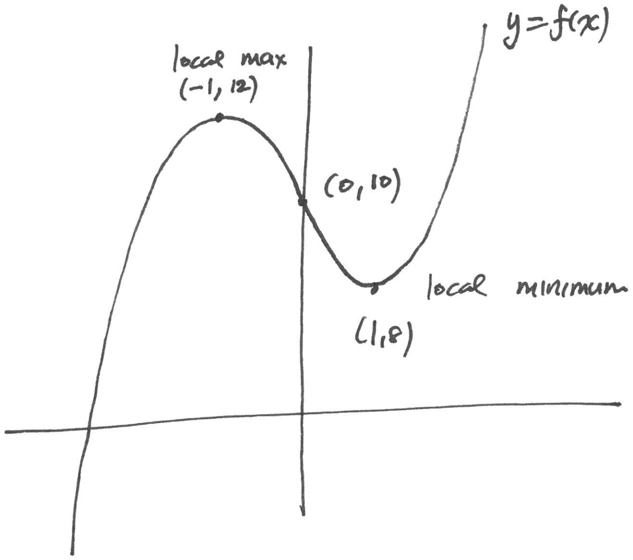
No inflection points

13. Critical points $x = 1$ or $x = -1$

Inflection point $x = 0$

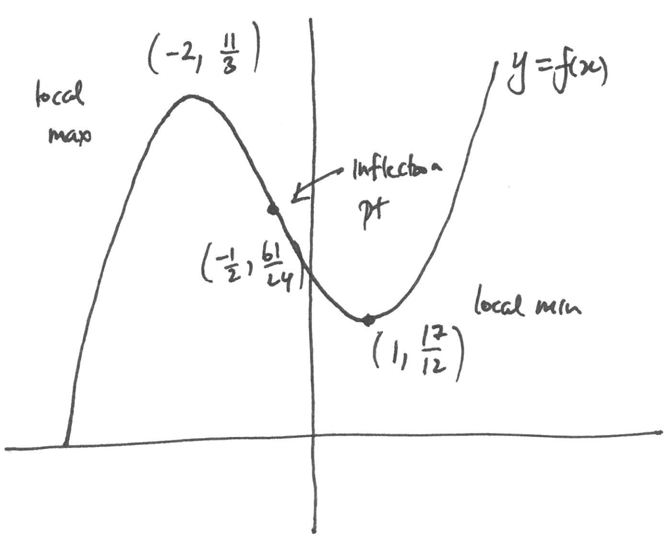
$x = 1$ (local min) ~~at $f(1)$~~

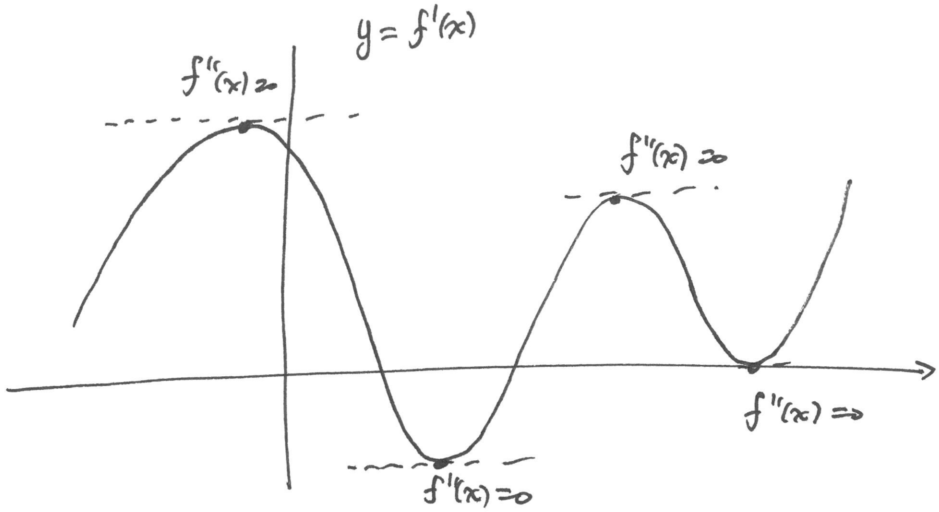
$x = -1$ (local max) ~~at $f(-1)$~~



15 Critical points $x = 1$, $x = -2$

Inflection points $x = -\frac{1}{2}$

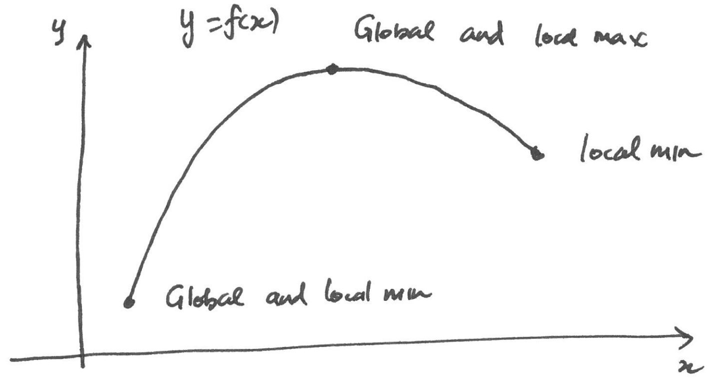




Since this is the graph of $y = f(x)$, the inflection points are the points where the slope of $y = f'(x)$ $[\,f''(x) = 0\,]$

4.3 1, 3, 8, 16, 17, 18, 27, ~~38, 39~~, 42

1.



3. (a) (IV)

(b) (I)

(c) III

(d) II

8. TRUE, If the maximum is not an endpoint, then it must be a critical point of f. BUT $x=0$ is ~~a~~ the only critical point of $f(x) = x^2$ and it is a min.

16. (a) $f'(x) = 3x^2 - 6x$, $f''(x) = 6x - 6$

(b) $x = 0$ and $x = 2$ are critical points of f.

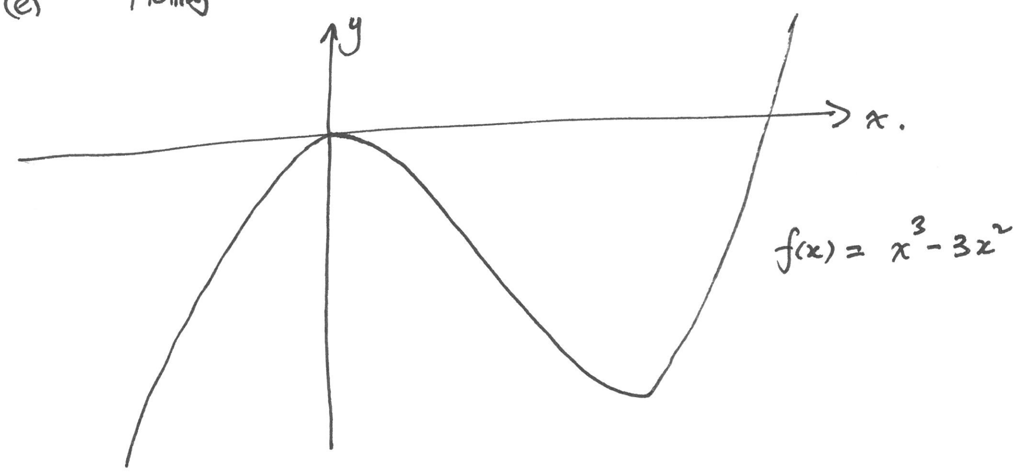
(c) $x = 1$ is an inflection point of f

(d) $f(-1) = -4$ $\quad f(0) = 0$ $\quad f(2) = -4,$ $f(3) = 0$

Global max at $x = 0$ is 0

Global min at $x = -1$ and $x = 2$ of -4.

(e) Plotting



17. (a) $f'(x) = 6x^2 - 18x + 12$

$f''(x) = 12x - 18$

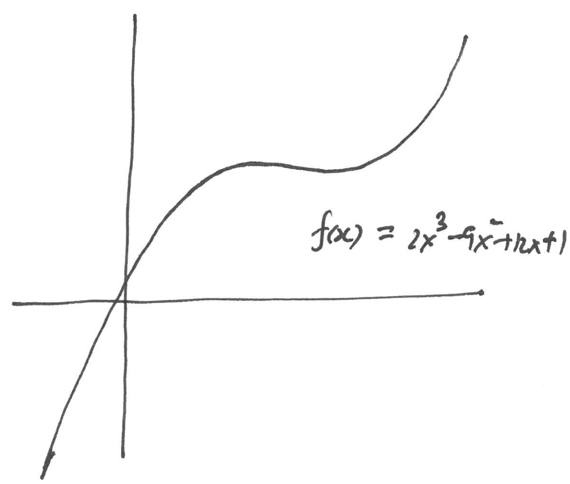
(b) $x = 1, 2$ are critical points

(c) $x = \frac{3}{2}$ is an inflection point

(d) $f(-0.5) = -7.5$, $f(3) = 10$, $f(1) = 6$, $f(2) = 5$

Global max at $x = 3$ is 10

Global min at $x = -0.5$ is -7.5



18. (a) $f'(x) = 3x^2 - 6x - 9$

$f''(x) = 6x - 6$

(b) $x = -1, 3$ are critical points

(c) $x = 1$ is an inflection point

(d) $f(-5) = -140$, $f(4) = -5$, $f(-1) = 2$, $f(3) = -12$

So Global max at $x = -1$ of 2

Global min at $x = -5$ of -140

27. $g'(x) = 4 - 2x$

$g'(x) = 0 \Rightarrow x = 2.$, x is a local maximum since $g''(2) = -2 < 0$

~~26~~ 42 (a) $q(0) = 0$

(b) Maximum value of $q(t)$ occurs when $q'(t) = 0$

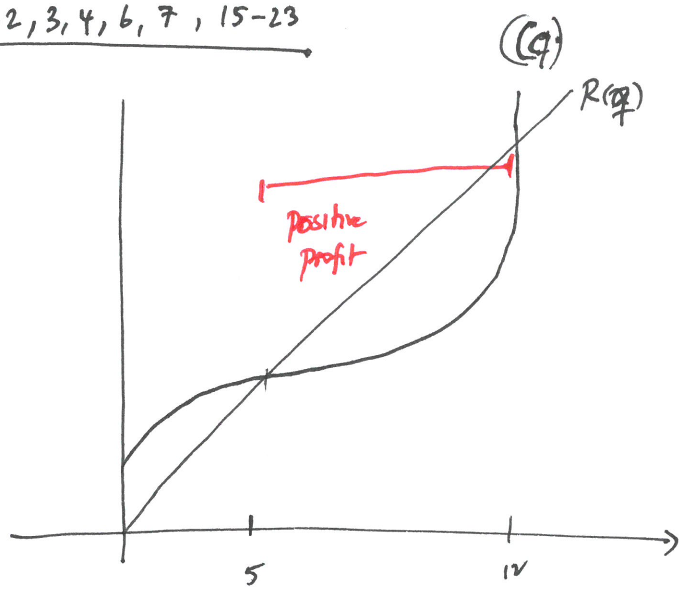
$$q'(t) = 20(-e^{-t} + 2e^{-2t}) = 0$$

$$\frac{-e^{-t}}{-e^{-2t}} = \frac{-2e^{-2t}}{e^{-2t}}$$

$$t = \ln(2) = 0.69 \text{ hrs}$$

4.4 1, 2, 3, 4, 6, 7, 15–23

1.



2.

$$\pi(q) = R(q) - C(q)$$
$$= 500q - q^2 - (150 + 10q)$$
$$= 490q - q^2 - 150$$

Maximum profit occurs when $\pi'(q) = 490 - 2q = 0$ so $q = 245$ items.

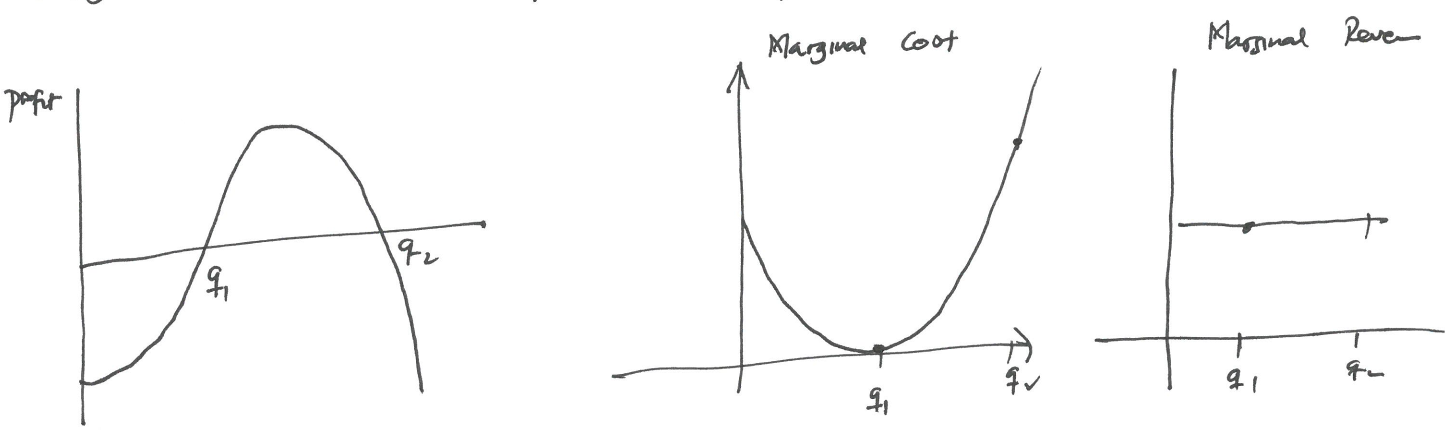
since $\pi''(q) = -2$, this is a max.

3.

$$MR = R'(q) = 450$$
$$MC = C'(q) = 6q$$

setting $MR = MC$ yields $6q = 450 \Rightarrow q = 75$ units. (Max profit)

4.



6 (a) $C(0)$ represents fixed costs before production

(b) Marginal cost decreases slowly, then increases as q increases

(c) Concave down implies decreasing marginal cost, while concave up implies increasing marginal cost

(d) An inflection point on the cost function is (locally) a point of minimum or maximum marginal cost

(e) The more items you produce the less it costs to produce extra items up to a point.

7 (a) $\pi'(50) = \$9.$

(b) $\pi'(90) = R'(90) - C'(90) = -\$3$

(c) If $R'(78) > C'(78)$ producing the 79^{th} item will increase profits.

~~MAH~~

15. Since MR > MC @ $q = 2000$, increase production

16. (a) MC at $q = 400$ is the slope of the tangent line to $C(q)$ at $q = 400$. We can estimate from the graph that this is roughly 1.

(b) Since MC > MR at $q = 500$, we estimate loss of profit.

(c) Maximum profit occurs when MR = MC, around $q = 400$.

17. MR = $0.20/item

18. $R(q)$ has a maximum at $q = 2250$, testing the endpoints $R(0) = 0$ and $R(2250) = \$50{,}625$, we conclude that the Revenue is max at $q = 2250$

19 (a) Revenue $= \$50{,}000$

(b) $R(q) = 70q - 0.02q^2$

(c) $q = 1750$ maximizes revenue

(d) Optimal price is $\$35$

(e) Maximum Revenue is $\$61{,}250$.

20. Revenue is maximized at a price of $\$4.50$. Quantity sold is 3600 and total Revenue is $\$16{,}200$

21 $R(p) = 2100p - 75p^2$

max occurs at $p = 14$.

22 (a) $\pi = R - C$

$$= (-5q + 4000)q - (6q + 5)$$

(b) Maximum profit occurs at $q = 399.4$.

(c) Max profit $= \pi(399.4) = \$797{,}596.80$

23 (a) ~~C = Fixed costs + Variable costs = 10,000 + 2q~~

~~(b) Demand~~

4.5 1-4, 6, 8, 9, 10

1(a) Since the graph is concave down, average cost gets smaller as q increases.

(b) The average cost is minimized at some point q for which the line through $(0,0)$ and $(q, C(q))$ is tangent to the cost curve.

2(a) \$1.60/unit

2(b) $a(q) = \frac{C(q)}{q}$

2(c) $q \cong 18{,}000$

3 (a) (i) $q \simeq$ \$8/unit

(ii) $C'(25) \cong$ \$4 per unit

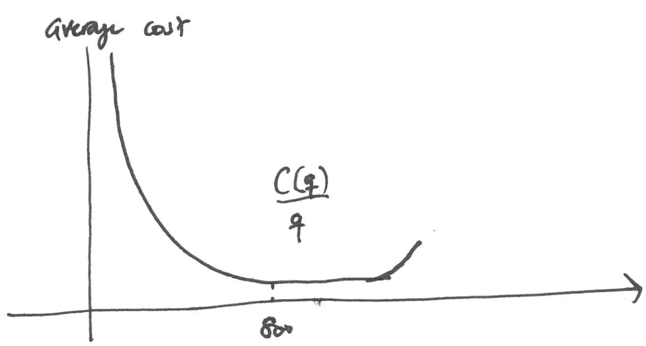
3(b) $q \cong 30$ units

4. (a) \$12

(b) Average cost \$37 $(q=100)$

@ $q = 1000$, average cost is \$14.50

6.



8. (a) Profit = \$21600

(b) Additional pair cost \$3 and can be sold for \$20 so this increases profits

(c) Increase production because this increases profits

9 (a) ~~$g(q) \neq \frac{(a)(q)}{q}$~~ so $C(q) = a(q) \cdot q$

$= 0.01q^3 - 0.6q^2 + 13q$

$a(q) = \frac{C(q)}{q}$

(b) $C'(q) = MC(q) = 0.03q^2 - 1.2q + 13.$ Minimum marginal cost is $MC(20) = 1$

$MC'(q) = 0.06q - 12 = 0 \Rightarrow q = 20$ so

(c) Minimum average cost

$a'(q) = 0.02q - 0.6$

$0.02q - 0.6 = 0 \Rightarrow q = 30$

Minimum average cost $= a(30) = \$4/\text{item}.$

(d) Marginal cost at $q = 30 = MC(30) = 4.$ This is the same as the minimal average cost.

10. (a) Marginal cost tells us that each additional product costs \$10, which is below average cost so producing more decrease average cost

(b) It is impossible to tell, $\Pi = R - C$, we need more information.