Name:
Homework 2 solutions
Math 151, Applied Calculus, Fall 2016

Note: for solutions to odd numbered problems - see the text.
Section 1.4

4 We know that the fixed cost is the cost that the company would have to pay if no items were produced. Thus Fixed cost

$$C(0) = \$5000$$

We know that the cost function is linear and we know that the slope of the function is exactly the marginal cost. Thus

$$\text{Slope} = \text{Marginal cost} = \frac{5020 - 5000}{5 - 0} = \frac{20}{5} = 4 \text{ dollars per unit produced}$$

Thus, the marginal cost is 4 dollars per unit produced. We know that since $C(q)$ is linear

$$C(q) = m \cdot q + b$$

where m is the slope and b is the value of $C(0)$, the vertical intercept. Or in other words, m is equal to the marginal cost and b is equal to the fixed cost. Thus

$$C(q) = 4q + 50000$$

7 (a) The statement $f(12) = 60$ says that when $p = 12$, we have $q = 60$. When the price is \$12, we expect to sell 60 units.

(b) Decreasing, because as price increases, we expect less to be sold.

8. a). The cost of producing 500 units is

$$C(500) = 6000 + 10(500) = 6000 + 5000 = \$11,000.$$

The revenue the company makes by selling 500 units is

$$R(500) = 12(500) = \$6000.$$

Thus, the cost of making 500 units is greater than the money the company will make by selling the 500 units, so the company does not make a profit. The cost of producing 5000 units is

$$C(5000) = 6000 + 10(5000) = 6000 + 50000 = \$56,000.$$

The revenue the company makes by selling 5000 units is

$$R(5000) = 12(5000) = \$60,000.$$

Thus, the cost of making 5000 units is less than the money the company will make by selling the 5000 units, so the company does make a profit.

b). The break-even point is the number of units that the company has to produce so that in selling those units, it makes as much as it spent on producing them. That is, we are looking for q such that

$$C(q) = R(q)$$

Solving for q gives $q = 3000$.

15. (a) We know that the cost function will be of the form

$$C(q) = b + m \cdot q$$

where m is the slope of the graph and b is the vertical intercept. We also know that the fixed cost is the vertical intercept and the variable cost is the slope. Thus, we have

$$C(q) = 5000 + 30q.$$

We know that the revenue function will take on the form

$$R(q) = pq$$

where p is the price charged per unit. In our case the company sells the chairs at \$50 a piece so

$$R(q) = 50q$$

(b) Marginal cost is \$30 per chair. Marginal revenue is \$50 per chair

(c) We know that the break-even point is the number of chairs that the company has to sell so that the revenue will equal the cost of producing these chairs. In other words, we are looking for q such that

$$C(q) = R(q)$$

Solving we get $q = 250$. Thus the break-even point is 250 chairs and \$12,500. Graphically this is the point of intersection of the cost and revenue functions.

18. a). $C(0) = 4000 + 2(0) = \$4000$

b). \$2 is the marginal cost

c). $p = \$10$.

d). Break even point is 500.

26. (a) Since cost is less than revenue for quantities in the table between 20 and 60 units, production appears to be profitable between those values.

(b) Profit = Revenue - Cost. You should obtain maximum profit at a level of production of 40 units.

36. $80b + 20s = 2000$, where b is the number of books bought and s is the number of social outings.

39. The original demand equation, $q = 100 - 5p$ tells us that

$$\text{Quantity demanded} = 100 - 5\big(Amountpaidbyconsumers\big)$$

The consumers pay $p + 2$ dollars per unit because they pay the price p plus \$2 tax. Thus, the new demand equation is

$$q = 100 - 5(p + 2) = 90 - 5p$$

40. The new supply equation is $4(p - 2) - 20 = 4p - 28$

42. a). The equilibrium price is \$100 and the quantity is 500.

b). If a \$6 tax is imposed on the suppliers the supply equation becomes

$$q = 10(p - 6) - 500 = 10p - 560.$$

c). To obtain the portion of tax paid by the consumer, find the new equilibrium price and quantity ($\$102, 460$). This means that the consumers pay \$2. The producer will pay \$4 which means they get to keep $\$102 - \$4 = \$96$ per item sold.

d). The total revenue for the government is Tax $\cdot$ Quantity sold $= 6 \cdot 460 = \$2760$.

Section 1.5

2. In each of these cases recall the t if

$$A = A_0(a)^t$$

then the initial amount is A_0 and if the growth factor $a > 1$ we have exponential growth otherwise we have decay.

4. $y = 30(0.94)^t$

8. a). Since the price is decreasing at a constant absolute rate, the price of the product is a linear function of t. In t days, the product will cost $804t$ dollars.

   b). Since we have a constant percentage of decrease the price of the product is $80(0.95)^t$ dollars.

10. a). 1.26%.

    b). In 2004 population is 6.4 billion. In 2010, population is 6.9 billion.

    c). Average rate of change is 0.083 billion people per year.

20. a). $a = 1.5$, $P_0 = 22.222$.

    b). growing at 50% annually.

28. a). $W = 39,295 + 16,321.6t$

    b). $W = 39,295(1.252)^t$.

    d). Plug in $t = 6$ into the linear and exponential models for $W(t)$.

30. For each table check the rate of change and percentage change. If the rate of change is constant the function if linear, if the percentage change is constant, the function is exponential. Otherwise neither.

32. The minimum wage has grown by 4.69% per year.

Section 1.6 – 2,10,18

2. $t = \dfrac{\ln 7}{\ln 5} \approx 1.209$

10. $t = 2\ln\left(\frac{5}{3}\right) \approx 1.0217$

18. Initial quantity = 7.7; growth rate = $-0.08 = -8\%$ (decay).

29. We use the information to create two equations: $P_0e^{3k} = 140$ and $P_0e^{1k} = 100$.

    (a) Divide the two equations and solve for k

$$\begin{aligned} \frac{P_0e^{3k}}{P_0e^k} &= \frac{140}{100} \\ e^{2k} &= 1.4 \\ 2k &= \ln 1.4 \\ k &= 0.5\ln 1.4 \approx 0.168 \end{aligned}$$

(b) You can use either equation to find the value of P_0. Using the first equation we have:

$$P_0 = \frac{140}{e^{0.504}} = 84.575$$

(c) The instal quantity is 84.575 and the quantity is growing at a continuous rate of 16.8% per unit time.

36 (a) i. $P = 1000(1.05)^t$

ii. $P = 1000e^{0.05t}$

(b) i. 1629

ii. 1649

41 (a) Since the percent rate of growth is constant and given as a continuous rate, we use the exponential function $S = 6.1e^{0.042t}$

(b) In 2015, we have $t = 4$ and $S = 6.1e^{0.042 \cdot 4} = 7.21$ billion dollars.

(c) Using the graph we see that $S = 8$ at approximately $t = 6.5$. Solving $6.1e^{0.042t} = 8$ gives $t = 6.456$ years.

42 (a) Current revenue is $R(0) = 5$ million dollars. In 2 years the revenue will be $R(2) = 5e^{-0.15(2)} = 3.704$ million dollars.

(b) $t = 4.108$. The revenue will have fallen to \$2.7 million early in the fifth year.