Name:
Homework 2 solutions
Math 151, Applied Calculus, Spring 2016

Section 1.7 – 1,2,3,10,22,28,33,35,40

1 In both cases the initial deposit was \$20. Compounding continuously earns more interest than compounding annually at the same interest rate. Therefore, curve A corresponds to the account which compounds interest continuously and curve B corresponds to the account which compounds interest annually. We know that this is the case because curve A is higher than curve B over the interval, implying that bank account A is growing faster, and thus is earning more money over the same time period.

2 $C = 2$, the intial amount, $\alpha = -\ln(2)$ so that $y(2) = 2e^{2(\ln 2} = 0.5$.

3 (a) If the interest is added only once a year (i.e. annually), then at the end of a year we have $1.055x$ where x is the amount we had at the beginning of the year. After two years, well have $1.055(1.055x)$ and after eight years, well have $(1.055)^8x$. Since we started with \$1000, after eight years well have $(1.055)^8(1000) \approx \$1534.69$.

(b) If an initial deposit of P_0 is compounded continuously at interest rate r then the account will have $P = P_0e^{rt} = 1000e^{(0.055)(8)} \approx \$1552.71$

10 (a) We have a continuous rate, therefore $W = 18,000e^{0.27t}$.

(b) $t = 9.745$

22 We have $P_0 = 500$, so $P = 500e^{kt}$. We can use the fact that $P = 1,500$ when $t = 2$ to find k:

$$\begin{aligned} 1,500 &= 500ekt \\ 3 &= e2k \\ \ln 3 &= 2k \end{aligned}$$

Solving for k yeilds $k = \dfrac{\ln 3}{2} \approx 0.5493$. The size of the population is given by

$$P = 500e^{0.5493t}$$

At $t = 5$, we have

$$P = 500e^{0.5493(5)} \approx 7794$$

28 We know that the world population is an exponential function over time. Thus the function for world population will be of the form

$$P(t) = P_0e^{rt}$$

where P_0 is the initial population, t is measured in years after 2012 and r is the continuous rate of change. We know that the initial population is the population in the year 2012 so

$$P_0 = 7\text{billion}$$

We are also told that in the year 2025 the population will be 8 billion, that is

$$P(13) = 8\text{billion}.$$

Solving for r we get

$$r = \frac{\ln \frac{8}{7}}{13} \approx 0.010272$$

Thus, we know that the continuous rate r is 1.02722%. The annual growth rate a is

$$a = e^{0.010272} - 1 \approx 0.010325$$

Or in other words, the annual growth rate is projected to be 1.0325%.

33 Future value $= 10,000e^{(0.03)(8)} = \$12,712.49$.

35 Present value $= \$6549.85$.

40 We compare the future values of the lump sum

$$2,800e^{0.06(2)} = \$3,156.99$$

and the installments

$$1,000e^{0.06(2)} + 1,000e^{0.06(1)} + 1,000 = 3,189.33$$

therefore it is clear that in this case, the installments result in a higher future value, so we take the installment option.