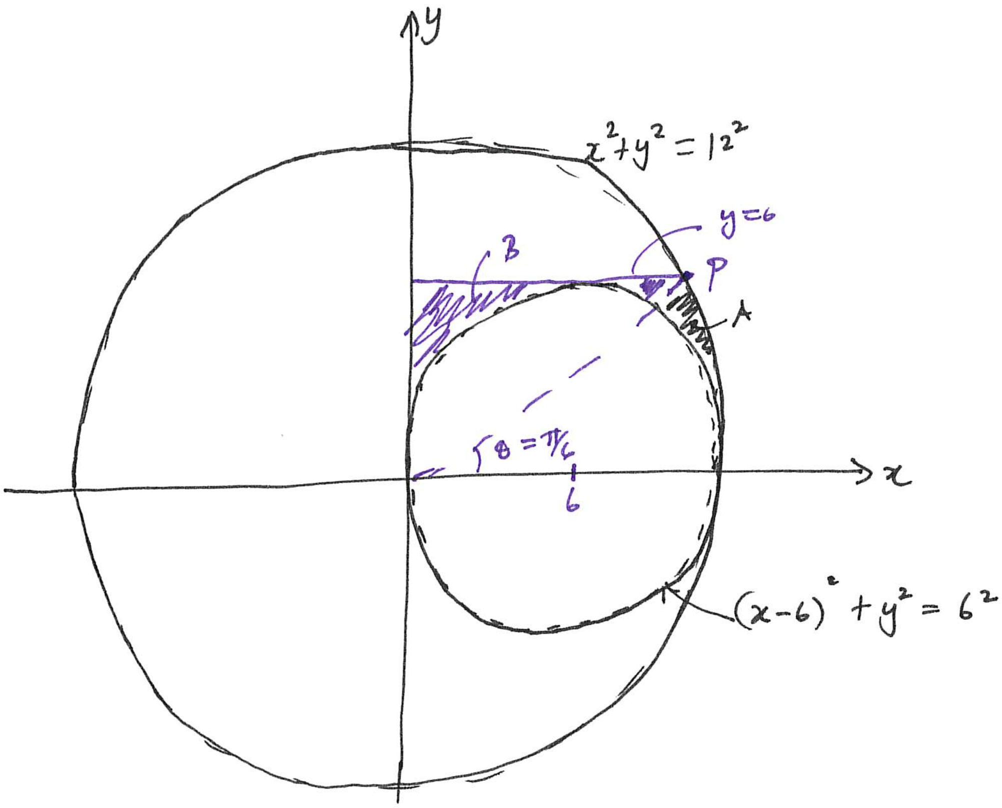
EXTRA CREDIT PROBLEM



$P = (6\sqrt{3}, 6)$

so $\theta = \tan^{-1}\left(\frac{6}{6\sqrt{3}}\right) = \frac{\pi}{6}$

Break up the area into 2 regions A & B (see above)

$$A = \int_0^{\frac{\pi}{6}} \int_{12\cos\theta}^{12} r\,dr\,d\theta = \int_0^{\frac{\pi}{6}} \frac{r^2}{2}\Big|_{12\cos\theta}^{12}$$

$$= \frac{12^2}{2}\int_0^{\frac{\pi}{6}} 1 - \cos^2\theta\,d\theta = 72\int_0^{\frac{\pi}{6}} \sin^2\theta\,d\theta$$

$$= \frac{72}{2}\int_0^{\frac{\pi}{6}} (1-\cos 2\theta)\,d\theta = 36\left(\theta - \frac{1}{2}\sin(2\theta)\right)\Big|_0^{\frac{\pi}{6}}$$

$$= 36\left(\frac{\pi}{6} - \frac{1}{2}\sin\left(\frac{\pi}{3}\right)\right)$$

$$= 6\pi - 18\cdot\frac{\sqrt{3}}{2}$$

$$= 6\pi - 9\sqrt{3}$$

Note that the upper limit in r for the region B is $y=6$

we have to convert this to polar! Indeed, $y = r\sin\theta \Rightarrow 6 = r\sin\theta$ so $r = \frac{6}{\sin\theta}$.

$$B = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \int_{12\cos\theta}^{\frac{6}{\sin\theta}} r\,dr\,d\theta$$

$$\int_{12\cos\theta}^{\frac{6}{\sin\theta}} r\,dr = \left.\frac{r^2}{2}\right|_{12\cos\theta}^{\frac{6}{\sin\theta}} = 18\csc^2\theta - 72\cos^2\theta$$

so

$$B = 18\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \csc^2\theta\,d\theta - 72\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta\,d\theta$$

$$\left.-18\cot\theta\right|_{\frac{\pi}{6}}^{\frac{\pi}{2}} = 18\sqrt{3}$$

$$\frac{72}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (1+\cos 2\theta)\,d\theta$$

$$= 36\left.\left(\theta + \frac{1}{2}\sin(2\theta)\right)\right|_{\frac{\pi}{6}}^{\frac{\pi}{2}}$$

$$= 36\left[\frac{\pi}{2} - \left(\frac{\pi}{6} + \frac{1}{2}\frac{\sqrt{3}}{2}\right)\right]$$

$$= 36\left[\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right]$$

$$= 12\pi - 9\sqrt{3}$$

so

$$B = 18\sqrt{3} - (12\pi - 9\sqrt{3})$$

$$= 27\sqrt{3} - 12\pi$$

so total area $= (6\pi - 9\sqrt{3}) + (27\sqrt{3} - 12\pi)$

$$= 18\sqrt{3} - 6\pi$$