MA 428: Homework 1: Numerical Differentiation (due Monday September 25, 2017)

Theoretical

1. Explain the difference between polynomial *approximation* and *interpolation* of a function f.

2. Problem 5 on page 226.

3. (a) Derive the difference formula

$$f'(x_0) \approx \frac{f(x_0 + 2h) - f(x_0 - h)}{3h}$$

   for approximating the first derivative by defining an interpolant of f at the points $x_0 - h$ and $x_0 + 2h$ then differentiating the interpolant.

   (b) What is the error term associated with the formula?

   (c) Estimate the value of h that results in the lowest error for the method.

4. (a) Derive the following backward difference approximation for the second derivative

$$f''(x) \approx \frac{f(x - 2h) - 2f(x - h) + f(x)}{h^2}$$

   (b) What is the error term associated with the formula?

   (c) Determine the optimal value of h that achieves the lowest error for the method.

5. Problem 6 on page 226.
   HINT: Find the Taylor expansions (do 5 terms including the error term) of $f(x + h)$ and $f(x + 2h)$ about x then combine the series as

$$Af(x) + Bf(x + h) + Cf(x + 2h)$$

   Group terms involving $f(x), f'(x), \ldots$. Notice that in order to approximate $f''(x)$ you will need to find A, B and C such that coefficients of $f(x)$ and $f'(x)$ are both zero and the coefficient of $f''(x)$ is 1. Use these 3 conditions to set up a system of 3 equations and solve for A, B and C.

Computational

In the following exercises you may use (with appropriate modifications) the provided codes

`numerical_diff.m, run_numerical_diff.m`

1. Verify numerically using the function $f(x) = \ln(x)$ and $x_0 = 1$ the convergence rates of the numerical differentiation formulas from Problems $2-5$ by computing the approximate derivative for a decreasing sequence of values of h. In addition, verify that the theoretical optimal values of h you estimated in $3(c)$ and $4(c)$ are consistent with the observed values.

Submission

Email your zipped m files, including your **summary file** with a discussion of your results to `pchidyagwai@loyola.edu` with email heading MA428_HWn, where n is the assignment number.