

## MA 428: Homework 3: Euler's Method

Due: Monday, October 30

### Theoretical

1. Page 295, Number 2.
2. Define the following terms in the context of initial value problems
  - (a) One-step method
  - (b) Local truncation error
  - (c) Global error
  - (d) Consistent method
3. Write down the result of applying one step of Euler's method using step size  $h = 0.1$  to the following equations

(a)

$$y'(t) = (t + 1)e^{y(t)}, \quad y(0) = 0$$

(b)

$$R'(t) = (2 - F(t))R(t), \tag{1}$$

$$F'(t) = (R(t) - 2)F(t) \tag{2}$$

starting with  $R_0 = 2$  and  $F_0 = 1$ .

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### Computational

1. In class we have discussed Euler's method for solving the initial value problem

$$y'(t) = f(t, y(t)), \quad y(a) = y_0, \quad a \leq t \leq b. \tag{3}$$

The method starts with an initial value  $y_0$  and marches through with  $N$  time steps of size  $h$  on the interval  $[a, b]$  as follows

$$y_{k+1} = y_k + hf(t_k, y_k)$$

- (a) Write a function `approx_ode.m` that implements Euler's method with the following heading

```
function [global_error,t_vals,approx_sol] = approx_ode(f,a,b,initial_val,N)
```

where the function `f.m` contains the definition of the initial value problem (3). Your function should return the global error at `t=b`, `t_vals` (a vector of  $t_k, k = 0, 1, \dots, N$ ) and `approx_sol` (a vector containing the approximate solution  $y_k, k = 0, 1, \dots, N$ ).

2. Consider the initial value problem

$$\frac{dy}{dt} = -(1 + t + t^2) - (2t + 1)y - y^2, \quad (0 \leq t \leq 3), \quad y(0) = -\frac{1}{2}$$

The exact solution of this problem is  $y(t) = -t - \frac{1}{e^t + 1}$ .

- (a) Using your code, run Euler's method to march from  $y = \text{initial\_val}$  for each of the number of steps  $N$  in the table below. In addition compute the error as the absolute value of the difference between your approximate solution and exact solution at  $t=3$  and compute the ratios between successive errors. The first line is meant as a guide to the expected format and to check the correctness of your code.

Euler's method				
N	Stepsize	Euler sol	Error	Ratio
10	0.3	-3.03548763	1.1938e-02	
20	0.15	-----	-----	-----
40	0.075	-----	-----	-----
80	0.0375	-----	-----	-----
160	0.01875	-----	-----	-----
320	0.009375	-----	-----	-----

- (b) Plot the true solution and your approximate solution on the same axes for  $N = 320$  with a legend to label each solution. Provide a .jpg plot of your solution.
- (c) Repeat problem (a), but change the initial condition to  $y(0) = -1.0$ . The exact solution in this case is  $y(t) = -t - 1$ .
- (d) Explain any difference in the rate of convergence between parts (a) and (c).

3. Consider the population model

$$\frac{dx}{dt} = rx \left( 1 - \frac{x}{k} \right) - \frac{x^2}{1+x^2}$$

The first term on the right-hand side is known as the logistic growth term. This term comes from the assumption that for small population levels, the population will grow at a rate proportional to the current level, while, for large populations, limited resources will cause the growth rate to decrease and eventually become negative. The parameters  $r$  and  $k$  are called the natural growth rate if the population and the environmental carrying capacity, respectively. The second term on the right-hand side represents harvesting/predation of the species by some other species (e.g. fish being caught by fisherman).

- (a) For  $r = 4$  and  $k = 20$ , use Euler's method to determine the eventual population level reached from an initial population of 2.44.  
 Note that to determine the eventual population, you will need to run Euler's method until the population stops changing, in this case run your code until the difference between your solution is less than  $1.0e-06$ .
- (b) Repeat part (a), but with the initial population changed to 2.40.
- (c) Provide a plot of the population in parts (a) and (b) and comment on your results.

4. Problem 3 on page 295.

## Submission

Email me your zipped m files, including your summary file with a discussion of your results for the computational part of the assignment. Your summary file must include all matlab output and answers to questions related to the output.