MA 428: Homework 5: Boundary value problems
Due: Tuesday, December 12

Theoretical

1. For the BVP

$$-u'' + (x^2 + 1)u = x^2, \quad 0 \leq x \leq 1 \tag{1}$$

on a computational grid with $h = \frac{1}{4}$. Write down the difference equations, using centred differences to approximate u'' for the following boundary conditions

   (a) $u(0) = \alpha, u(1) = \beta$.
   (b) $u(0) = \alpha, u'(1) = \gamma$.
   (c) $u'(0) = \kappa, u'(1) = \gamma$.

2. Page 376, Problem 1a.

3. Given the BVP

$$-u'' + u = f, \quad 0 \leq x \leq 1 \tag{2}$$
$$u(0) = u(1) = 0 \tag{3}$$

   (a) Derive the weak formulation for (2)–(3).
   (b) Derive the discrete problem resulting from the application of the finite element method using piecewise linear elements.

Computational

1. Use the provided code to solve the BVP

$$-u''(x) + r(x)u(x) = f(x), \quad 0 \leq x \leq 1$$
$$u(0) = \alpha, u(1) = \beta.$$

Where $r(x) = e^x$ and the boundary conditions and $f(x)$ are chosen in such a way that the true solution is $x^3 + cos(\pi x)$. Run your code on a computational grid with $n = 10, 20, 40, 80, 160$. Provide a plot (*.jpg) of the true solution along with the approximate solutions and provide a table of errors and ratios to confirm the second order convergence of the method.

2. Modify the code and repeat the above problem with mixed boundary conditions $u'(0) = \gamma, u(1) = \beta$.

Submission

Email me your zipped m files, including your summary file with a discussion of your results for the computational part of the assignment. Your summary file must include all matlab output and answers to questions related to the output.