12.4 SOLUTIONS

3. $\vec{a} \times \vec{b} = 14i + 4j + 2k$

7. $\vec{a} \times \vec{b} = (1-t)i - (t-t)j + (t^3 - t^2)k.$

8. $\vec{a} \times \vec{b} = 2i - j + k.$

20 $\pm \frac{1}{\sqrt{3}} (i - j - k)$

31 (a) Vector orthogonal to plane is $\langle 13, -14, 5 \rangle$

(b) Area of parallelogram $= \sqrt{390}$ so area of triangle $PQR = \frac{1}{2}\sqrt{390}$

33 Volume $= 9$

35 Volume $= 16.$

38 Compute $\vec{u} \cdot (\vec{v} \times \vec{w})$ and show that this quantity is ZERO so the vectors are coplanar.

43 $\tan\theta = \dfrac{|\vec{a} \times \vec{b}|}{\vec{a} \cdot \vec{b}} = \dfrac{3}{\sqrt{3}} = \sqrt{3} \Rightarrow \theta = 60°$

53 (a) No, $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} \Rightarrow \vec{a} \cdot (\vec{b} - \vec{c}) = 0$ so $\vec{a} \perp (\vec{b} - \vec{c})$, this can happen if $\vec{b} \neq \vec{c}$. Check $\vec{a} = \langle 1,1,1 \rangle$ $\vec{b} = \langle 1,0,0 \rangle$, $\vec{c} = \langle 0,1,0 \rangle$

(b) If $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ then $\vec{a} \times (\vec{b} - \vec{c}) = 0$

so $\vec{a}$ is parallel to $(\vec{b} - \vec{c})$

(c) Yes

Since $\vec{a} \cdot \vec{c} = \vec{a} \cdot \vec{b}$ then $\vec{a} \perp (\vec{b} - \vec{c})$

"perpendicular"

$\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ then $\vec{a}$ is parallel to $\vec{b} - \vec{c}$ from (b)

Since $\vec{a} \neq 0$ and it is both parallel ~~to~~ and perpendicular to $\vec{b} - \vec{c}$, so $\vec{b} - \vec{c} = \vec{0}$ so $\vec{b} = \vec{c}$.