14.2 SOLUTIONS TO EVEN PROBLEMS

8.
$$\lim_{(x,y)\to(3,2)} e^{\sqrt{2x-y}} = e^2.$$

14. $$\lim_{(x,y)\to(0,0)}\left(\frac{x^3-y^3}{x^2+xy+y^2}\right)$$

Notice that $x^3-y^3 = (x-y)(x^2+xy+y^2)$

so $$\lim_{(x,y)\to(0,0)}\left(\frac{x^3-y^3}{x^2+xy+y^2}\right) = \lim_{(x,y)\to(0,0)} \frac{(x-y)\cancel{(x^2+xy+y^2)}}{\cancel{(x^2+xy+y^2)}} = 0.$$

18. $$f(x,y) = \frac{xy^4}{x^2+y^8}$$

limit along the x-axis (set $y=0$)

$\lim_{x\to0} \frac{0}{x^2} = 0$ for $x\neq0$ so $f(x,y)\to0$ as $(x,y)\to(0,0)$

But along the curve $x=y^4$

$$\lim_{(x,y)\to(0,0)}\left(\frac{xy^4}{x^2+y^8}\right) = \lim_{(x,y)\to(0,0)}\left(\frac{y^4\cdot y^4}{y^8+y^8}\right) = \lim_{(x,y)\to(0,0)} \frac{y^8}{2y^8} = \frac{1}{2}$$

so the limit D.N.E.

40. Converting to polar using $r^2=x^2+y^2$

$$\lim_{(x,y)\to(0,0)} (x^2+y^2)\ln(x^2+y^2) = \lim_{r\to0^+} r^2\ln(r^2) = \lim_{r\to0} \frac{\ln(r^2)}{\left(\frac{1}{r^2}\right)} \overset{H}{=} 0$$

H = L'hopital's Rule!