20

$$\frac{\partial z}{\partial x} = xy\cos(xy) + \sin(xy)$$

$$\frac{\partial z}{\partial y} = x^2\cos(xy)$$

54

$$f_x(x,y) = a(ax+by)^{-1}$$

$$f_y(x,y) = b(ax+by)^{-1}$$

$$f_{xx} = -a(ax+by)^{-2}\cdot(a) = \frac{-a^2}{(ax+by)^2}$$

$$f_{xy} = \frac{-ab}{(ax+by)^2} = f_{yx}$$

$$f_{yy} = \frac{-b^2}{(ax+by)^2}$$

74. (a) If we fix y and allow x to very (f_x) the values on the level curves indicate that f decreases as we move through P in the positive x direction so $f_x < 0$

(b) f_y If we fix x and allow y to vary f increases as we move ~~forwards~~ through P in the positive y direction. so ($f_y > 0$)

(c) $f_{xx} = \frac{\partial}{\partial x}(f_x)$ so if we fix y and allow x to vary, f_{xx} is the rate of change of f_x as x increases. Note that for points to the right of P, the level curves are spaced farther apart (in x direction) compared to the left so f decreases less quickly w.r.t x to the right of P so $f_{xx} > 0$. (ie the negative value of f_x increases)

(d) $f_{xy} = \frac{d}{dy}(f_x)$

fix x and allow y to vary, f_{xy} is the rate of change of f_x as y increases.

Notice that the level curves are closer together in the x direction as y increases so f is decreasing more quickly w.r.t x for y values above P. $f_{xy} < 0$

(e) $f_{yy} > 0$.