Name: SOLUTIONS

Worksheet 4 - Double integrals

Math 351.01, Calculus III, Fall 2017

11/21/2017

Each problem is worth 5 points

1. $\int_0^1 \int_0^3 e^{x+3y}\, dx dy$

Note that $e^{x+3y} = (e^x)(e^{3y})$ so

$$\int_0^1 \int_0^3 e^{x+3y} dx dy = \left(\int_0^1 e^{3y} dy\right)\left(\int_0^3 e^x dx\right)$$

$$= \left(\frac{e^{3y}}{3}\Big|_{y=0}^{1}\right)\left(e^x\Big|_{x=0}^{3}\right) = \underline{\frac{1}{3}(e^3-1)(e^3-1)}$$

2. $\int_0^\pi \int_1^2 y \sin(xy)\, dx dy$

Inner integral

$$= \int_1^2 y \sin(xy) dx = -\cos(xy)\Big|_{x=1}^{2} = -\cos(2y) + \cos(y)$$

so $$\int_0^\pi \int_1^2 y\sin(xy) dx dy = \int_0^\pi (-\cos(2y) + \cos(y)) dy = -\frac{1}{2}\sin(2y) + \sin(y)\Big|_{y=0}^{\pi}$$

$$= 0!$$

Notice that #3 is the same integral as in #2 but the order is different. The difference in order is enough to make the integral much more complicated so be aware ORDER matters!

3. $\int_1^2 \int_0^\pi y \sin(xy)\, dy dx$

Inner integral $\int_0^\pi y \sin(xy)\, dy$

Integrate by parts
$u = y \quad dv = \sin(xy)dy$
$\frac{du}{dy} = 1 \quad v = -\frac{\cos(xy)}{x}$

$$= \frac{-y\cos(xy)}{x}\Big|_{y=0}^{\pi} + \frac{1}{x}\int_0^\pi \cos(xy)\, dy$$

$$= \frac{-\pi\cos(\pi x)}{x} + \frac{1}{x^2}\left(\sin(xy)\right)\Big|_{y=0}^{\pi} = \frac{-\pi\cos(\pi x)}{x} + \frac{\sin(\pi x)}{x^2}$$

Now for the outer integral

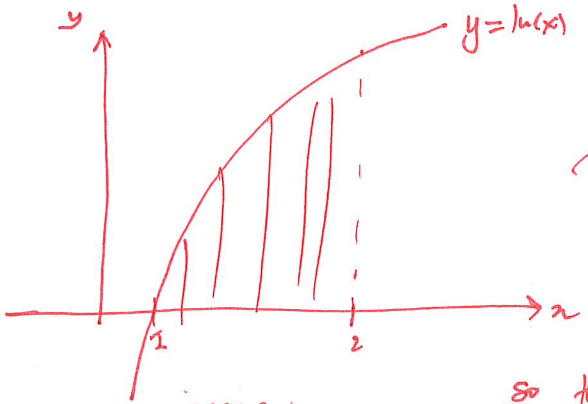
Notice that $\int \frac{-\pi\cos(\pi x)}{x} dx = \frac{-\sin(\pi x)}{x} - \int \frac{\sin(\pi x)}{x^2} dx$ so $\int\left(\frac{-\pi\cos(\pi x)}{x} + \frac{\sin(\pi x)}{x^2}\right)dx$

$= -\frac{\sin(\pi x)}{x}$

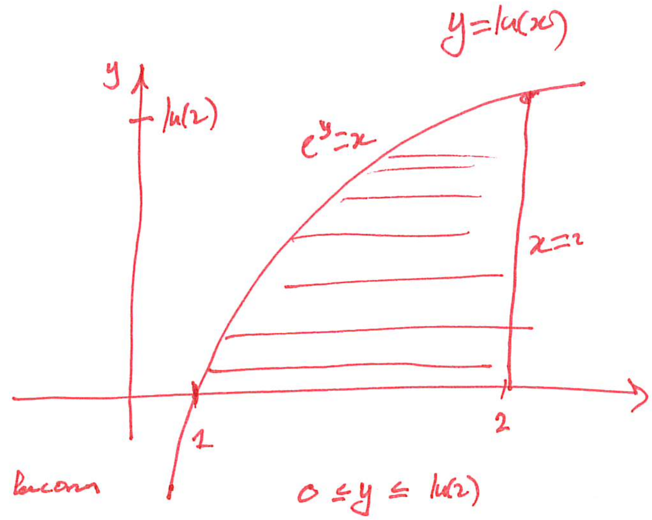
so $\int_1^2 \int_0^\pi y\sin(xy)\, dy dx = -\frac{\sin(\pi x)}{x}\Big|_{x=1}^{2} = 0!$

4. Sketch the region and switch the order of integration for $\int_1^2 \int_0^{\ln(x)} f(x,y)\,dydx$

As given, the ~~one~~ integral is over



switch to horizontal Region



$1 \le x \le 2$
$0 \le y \le \ln(x)$

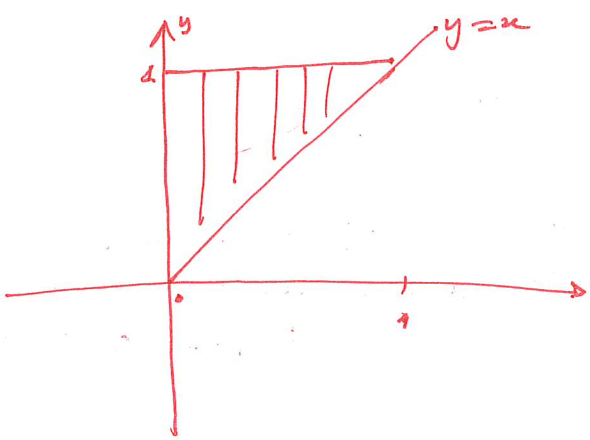
So the integral becomes

$$\int_0^{\ln(2)} \int_{e^y}^{2} f(x,y)\,dxdy$$

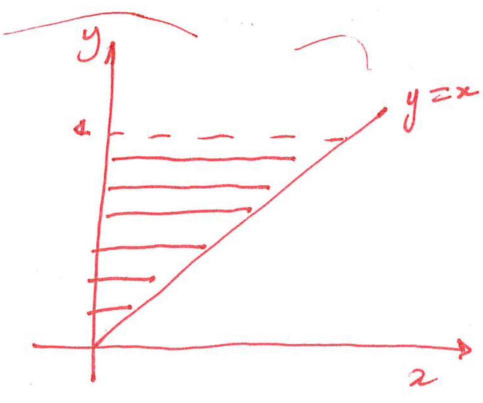
$0 \le y \le \ln(2)$
$e^y \le x \le 2$

5. Evaluate $\int_0^1 \int_{\cancel{x}}^{\cancel{1}} \sin(y^2)\,dydx$ by switching the order of integration.

As given, notice that is is impossible to integrate $\int \sin(y^2)\,dy$ so our best hope is to switch the order of integration.



switch to horizontal



$0 \le y \le 1$
$0 \le x \le y$

As given So the integral becomes

$$\int_0^1 \int_0^y \sin(y^2)\,dxdy$$

- Inner integral

$$\int_0^y \sin(y^2)\,dx = x\sin(y^2)\Big|_{x=0}^{y} = y\sin(y^2)$$

$$\int_0^1 \int_0^y \sin(y^2)\,dxdy = \int_0^1 y\sin(y^2)\,dy = \cancel{\ldots} -\frac{1}{2}\cos(y^2)\Big|_0^1 = \underline{\frac{1}{2}(1-\cos 1)}$$

$u = y^2$
$\frac{du}{dy} = 2y$
$\frac{1}{2}du = ydy$