Gaussian Quadrature

- We use the following formula with weights w_i and nodes x_i

$$\int_{-1}^{1} f(x)\,dx \approx \sum_{i=1}^{n} w_i f(x_i)$$

- The nodes and weights are of the form:

n (# of points)	x_i	w_i
2	0.5773502691896257	1.0000000000000000
	-0.5773502691896257	1.0000000000000000
3	0.7745966692414834	0.5555555555555556
	0	0.8888888888888888
	-0.7745966692414834	0.5555555555555556
4	0.8611363115940525	0.3478548451374544
	0.3399810435848563	0.6521451548625460
	-0.3399810435848563	0.6521451548625460
	-0.8611363115940525	0.3478548451374544
...	...	...

- These can be found in most Numerical analysis texts.

Matlab implementation

- We approximate an integral over $[a, b]$ as follows:

$$\int_a^b f\,dx = \sum_{i=1}^{n} \tilde{w}_i f(\tilde{x}_i)$$

where $\tilde{w}_i = w_i \frac{b-a}{2}$ and $\tilde{x}_i = \frac{(b+a) + x_i(b-a)}{2}$

- The function `lgwt` computes $\tilde{w}_i$ and $\tilde{x}_i$ for any n on an interval $[a, b]$.

```
[x,w]=lgwt(4,0,pi);
```

- We can then specify the function and approximate the integral

```
>> f=sin(x);
>> approx_int = sum(w.*f)
approx_int =
    1.999984228457722
```

Comparison to Composite Trapezoidal rule

- Gaussian Quadrature is far superior!

n	Trapeziodal Error	Gaussian Error
2	0.429203673205103	0.064180425348863
4	0.103881102062960	1.577154227838662e-05
8	0.025768398054449	4.440892098500626e-15
16	0.006429656227660	4.440892098500626e-16

All is not lost for Composite methods!

$$\int_0^{2\pi} e^{\sin(x)}\, dx$$

n	Trapeziodal Error	Gaussian Error
2	1.67174121383325e+00	1.52774484383558e+00
4	3.43969188091968e-02	2.02753056344147e-01
8	1.25168893738703e-06	1.22293544133538e-03
16	6.21724893790088e-15	4.724131308364576e-09