Identities & Basic integrals
MATH 351.01, CALCULUS III, FALL 2017

1. (a) $\sin^2 x + \cos^2 x \equiv 1$
   (b) $\sin^2 x \equiv \frac{1}{2}(1 - \cos(2x))$
   (c) $\cos^2 x \equiv \frac{1}{2}(1 + \cos(2x))$
   (d) $\cos^2 x - \sin^2 x \equiv \cos(2x)$
   (e) $\sin(2x) \equiv 2\sin x \cos x$
   (f) $\tan^2 x + 1 \equiv \sec^2 x$
2. (a) $\sin A \cos B \equiv \frac{1}{2}[\sin(A-B) + \sin(A+B)]$
   (b) $\sin A \sin B \equiv \frac{1}{2}[\cos(A-B) - \cos(A+B)]$
   (c) $\cos A \cos B \equiv \frac{1}{2}[\cos(A-B) + \cos(A+B)]$

Table of Basic Integrals

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C, \quad n \neq -1 \tag{1}$$

$$\int \frac{1}{x}\,dx = \ln|x| + C \tag{2}$$

$$\int e^x\,dx = e^x + C \tag{3}$$

$$\int a^x\,dx = \frac{1}{\ln a}a^x + C \tag{4}$$

$$\int \sin x\,dx = -\cos x + C \tag{5}$$

$$\int \cos x\,dx = \sin x + C \tag{6}$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C \tag{7}$$

$$\int \sec^2 x\,dx = \tan x + C \tag{8}$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C \tag{9}$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + C, \quad a > 0 \tag{10}$$

Some useful formulae

1.
$$\iint_D f(x,y)\,dA = \int_\alpha^\beta \int_a^b f(r\cos\theta, r\sin\theta)\,r dr d\theta$$
where
$$D = \{(r,\theta), \alpha \le \theta \le \beta, a \le r \le b\}$$

2.
$$\iiint_E f(x,y,z)\,dV = \int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} \int_{u_1(r\cos\theta, r\sin\theta)}^{u_2(r\cos\theta, r\sin\theta)} f(r\cos\theta, r\sin\theta, z) r\,dz dr d\theta$$
where
$$E = \{(r,\theta,z) | \alpha \le \theta \le \beta, h_1(\theta) \le r \le h_2(\theta), u_1(r\cos\theta, r\sin\theta) \le z \le u_2(r\cos\theta, r\sin\theta)\}$$

3.
$$\iiint_E f(x,y,z)\,dV = \int_c^d \int_\alpha^\beta \int_a^b f(\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi)\rho^2 \sin\phi\,d\rho d\theta d\phi$$
where
$$E = \{(\rho,\theta,\phi) | a \le \rho \le b, \alpha \le \theta \le \beta, c \le \phi \le d\}$$