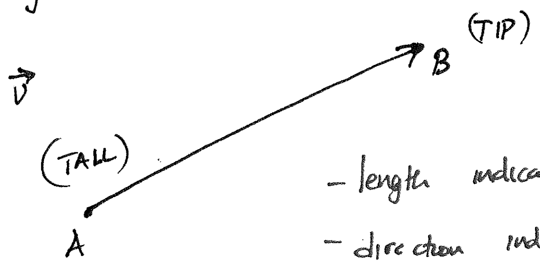
VECTORS

- definition of vectors
- addition and scalar multiplication
- unit vector,
- relationship between points and vectors.

Vector

Indicates A a quantity with both magnitude and direction.



- length indicates magnitude
- direction indicates direction.

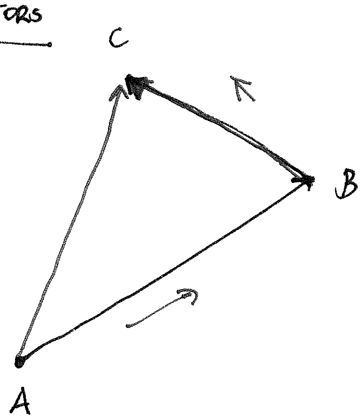
eg velocity, force, displacement.

A - initial point (TAIL)
B - terminal point (TIP)

NOTATION $\vec{v} = \overrightarrow{AB}$. , A and B are points in $\mathbb{R}^2$ or $\mathbb{R}^3$.

Combining Vectors

Motion



- particle moves from A to B

$\Rightarrow$ • displacement is $\overrightarrow{AB}$

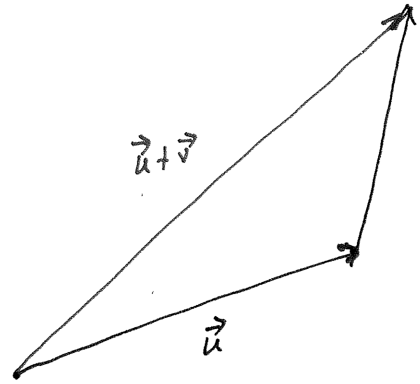
- moves from B to C

$\Rightarrow$ displacement is $\overrightarrow{BC}$

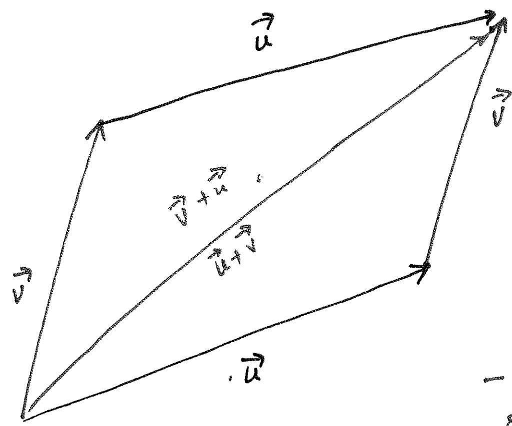
combined motion is A to C with a resulting displacement is $\overrightarrow{AC}$

$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$$

In general, if we start with $\vec{u}$ and $\vec{v}$ ~~so that the tail~~ and move $\vec{v}$ so that the tail of $\vec{v}$ corresponds to the tip of $\vec{u}$, then we can define $\vec{u} + \vec{v}$.



move the tail of $\vec{v}$ to the tip of $\vec{u}$.



Parallelogram Law.

$\vec{u}+\vec{v} = \vec{v}+\vec{u}$.

Another way of thinking about the sum
- If we place $\vec{u}$ and $\vec{v}$ so that they start at the same point, then $\vec{u}+\vec{v}$ lies along the diagonal of the parallelogram with sides $\vec{u}$ and $\vec{v}$.

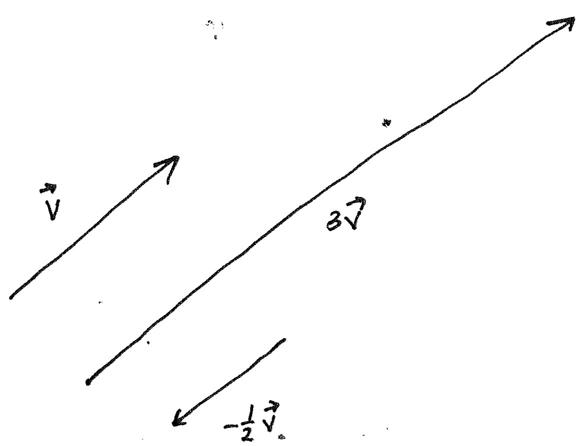
Scalar Multiplication

If c is a scalar and $\vec{v}$ is a vector then $c\vec{v}$ is a vector whose length is $|c|$ times the length of $\vec{v}$.

If $c>0$, the direction of $c\vec{v}$ is the same as $\vec{v}$

If $c<0$, the direction of $c\vec{v}$ is opposite to $\vec{v}$

Example



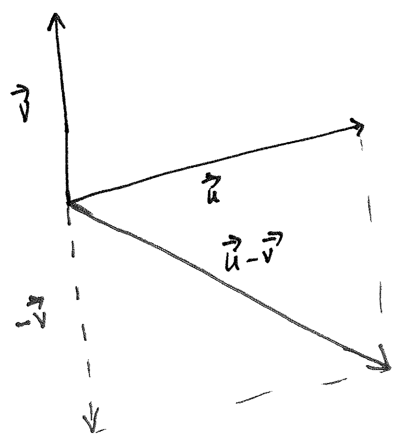
Remark

Two vectors are parrallel if they are scalar multiples of each other.

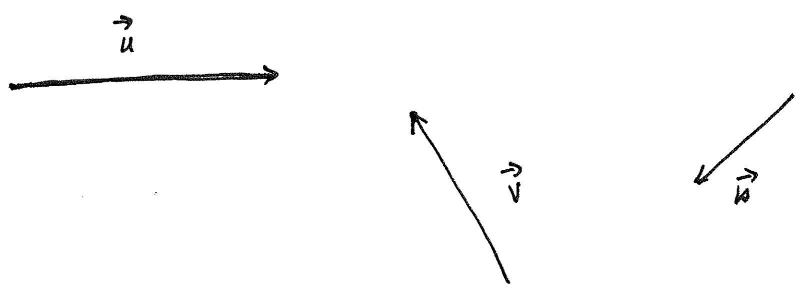
$\vec{v}_1 \parallel \vec{v}_2 \Rightarrow \vec{v}_1 = c\vec{v}_2$.

Subtracting vectors

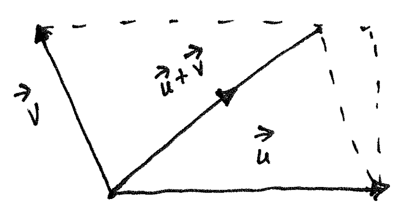
$$\vec{u} - \vec{v} = \vec{u} + (-\vec{v})$$



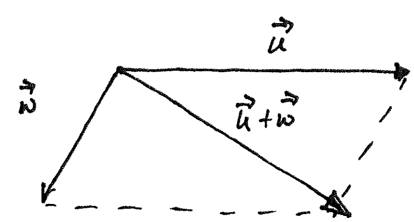
Example



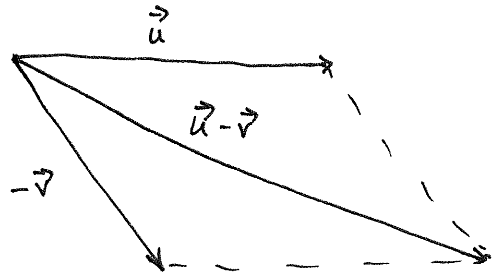
(i) $\vec{u} + \vec{v}$



(ii) $\vec{u} + \vec{w}$

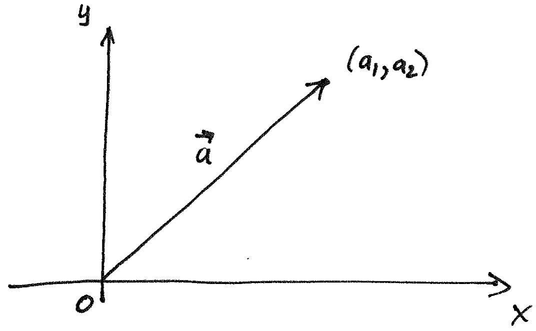


(iii) $\vec{u} - \vec{v} = \vec{u} + (-\vec{v})$

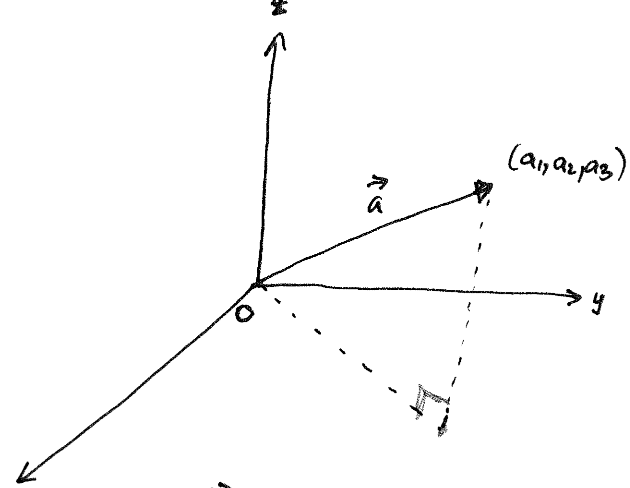


Components of vectors (Algebraic system for vectors)

If we place the tail of the vector at O and then tip at $a = (a_1, a_2)$ or $a = (a_1, a_2, a_3)$



$\vec{a} = \langle a_1, a_2 \rangle$



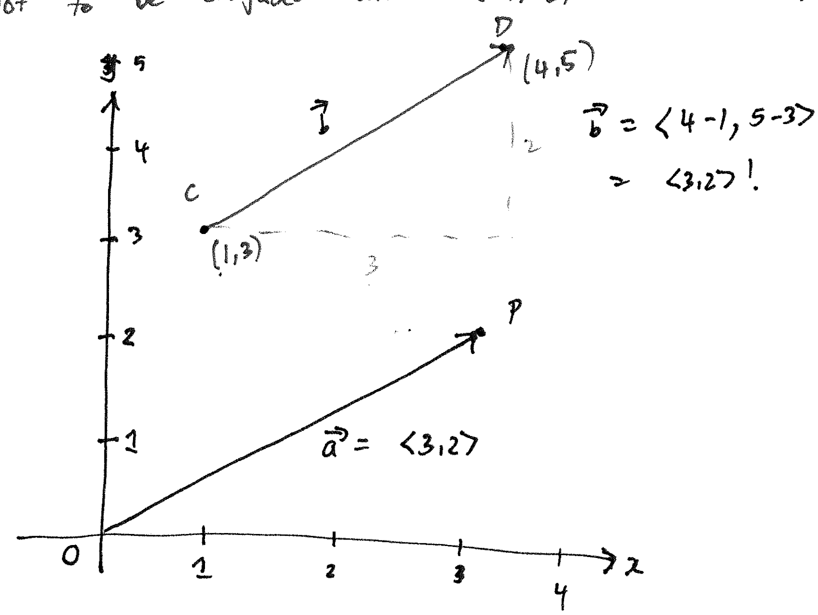
$\vec{a} = \langle a_1, a_2, a_3 \rangle$

The coordinates are called the components of $\vec{a}$.

~~We~~ Notation

We will use $\langle a_1, a_2 \rangle = \vec{a}$ ~~and~~ or $\langle a_1, a_2, a_3 \rangle = \vec{a}$ to denote vectors.

Not to be confused with (a_1, a_2) and (a_1, a_2, a_3) [points in 2D and 3D].



$\vec{b} = \langle 4-1, 5-3 \rangle$
$= \langle 3, 2 \rangle$!

$\vec{a} = \overrightarrow{OP}$

$\vec{a}$ is called the position vector of $\vec{P}$

$\vec{b} = \overrightarrow{CD}$ has the property move 3 in x and 2 in y

Infact, we say $\vec{b}$ is equivalent to $\vec{a}$.

In general, given 2 points

$A = (x_1, y_1, z_1)$ and $B = (x_2, y_2, z_2)$, ~~then~~ the vector $\vec{a} = \overrightarrow{AB}$ is

$$\vec{a} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle.$$

Magnitude / length / Norm

Notation

$|\vec{v}|$, or $||\vec{v}||$

In 2D, $||\vec{a}|| = \sqrt{a_1^2 + a_2^2}$

In 3D

$$||\vec{a}|| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

Properties of vectors

Let V_n be the set all n-dimensional vectors

$a \in V_n$ is of the form $\langle a_1, a_2, a_3, \ldots, a_n \rangle$.

Let α_1 and α_2 be scalars.

1. $\vec{a} + \vec{b} = \vec{b} + \vec{a}$
2. $\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$.
3. $\vec{a} + \vec{0} = \vec{a}$
4. $\vec{a} + (-\vec{a}) = 0$.
5. $\alpha_1(\vec{a} + \vec{b}) = \alpha_1 \vec{a} + \alpha_2 \vec{b}$
6. $(\alpha_1 + \alpha_2)\vec{a} = \alpha_1 \vec{a} + \alpha_2 \vec{a}$
7. $(\alpha_1 \alpha_2)\vec{a} = \alpha_1(\alpha_2 \vec{a})$
8. $1\vec{a} = \vec{a}$.

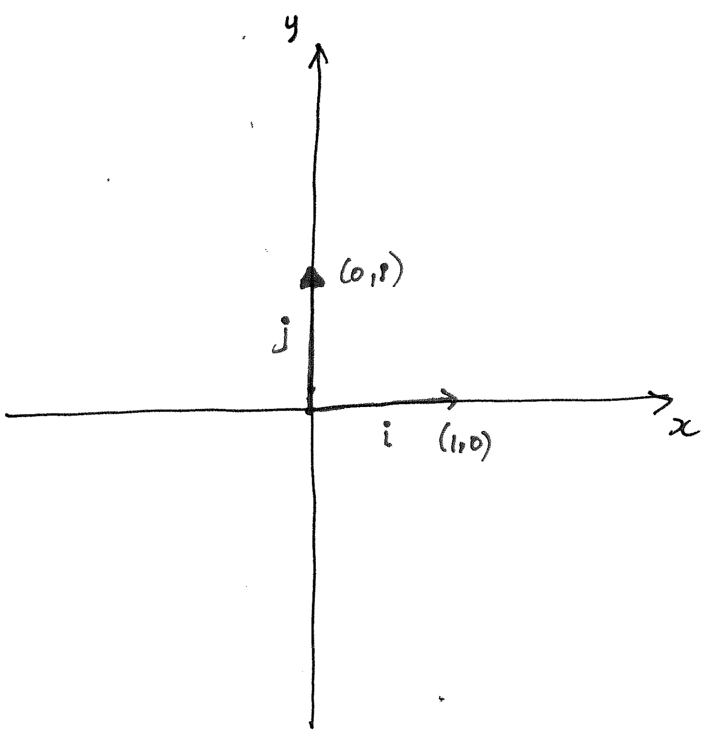
Standard Basis

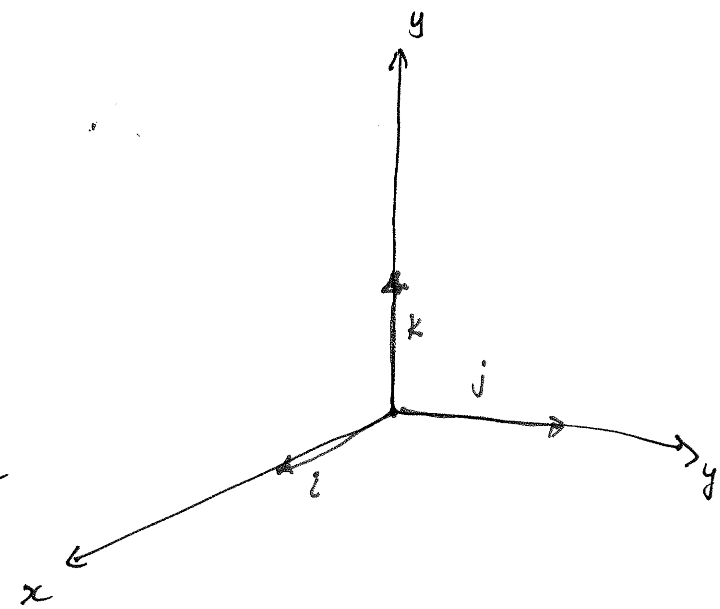
$$\vec{i} = \langle 1,0,0\rangle, \quad \vec{j} = \langle 0,1,0\rangle, \quad \vec{k} = \langle 0,0,1\rangle.$$

Let $\vec{a} = \langle a_1, a_2, a_3\rangle$, we can write $\vec{a}$ as a linear combination of i, j, k

$$\begin{aligned}\vec{a} = \langle a_1, a_2, a_3\rangle &= \langle a_1, 0, 0\rangle + \langle 0, a_2, 0\rangle + \langle 0, 0, a_3\rangle \\ &= a_1\langle 1,0,0\rangle + a_2\langle 0,1,0\rangle + a_3\langle 0,0,1\rangle \\ &= a_1 i + a_2 j + a_3 k\end{aligned}$$

Basis vectors





The unit vector is a vector whose length is 1

Examples of unit vectors? i, j, k

In general, if $\vec{a} \neq 0$, then the unit vector $\vec{u}$ points in the same direction as $\vec{a}$

$$\vec{u} = \frac{1}{|\vec{a}|}\vec{a} = \frac{\vec{a}}{|\vec{a}|}.$$