13.1 VECTOR VALUED FUNCTIONS

A vector valued function is one whose domain and range are vectors.

$$\vec{r}(t) = \langle f(t), g(t), h(t) \rangle = f(t)\,i + g(t)\,j + h(t)\,k$$

Example #1

1. $\vec{r}(t) = \langle \sqrt{4-t^2},\ e^{-3t},\ \ln(t+1) \rangle$

The domain of $\vec{r}(t)$ - all values of t for which ~~t~~ $f(t), g(t)$ and $h(t)$ are well defined.

$|t| < 2$

$f(t) = \sqrt{4-t^2}$, well defined for $4 - t^2 \geq 0 \Rightarrow 4 \geq t^2$, $\cancel{t \leq \pm 2}$

$g(t) = e^{-3t}$, well defined for $-\infty < t \leq \infty$

$h(t) = \ln(t+1)$, well defined for $t+1 > 0 \Rightarrow t > -1$

domain of $\vec{r}(t)$ is $(-1, 2]$.

FACT If $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$, then

$$\lim_{t \to a} \vec{r}(t) = \left\langle \lim_{t \to a} f(t),\ \lim_{t \to a} g(t),\ \lim_{t \to a} h(t) \right\rangle$$

Provided that the limit of the componend functions exist.

Example #2

$$\lim_{t \to 0} \left(e^{-3t} i + \frac{t^2}{\sin^2 t} j + \cos(2t)\,k \right)$$

$$= \left(\lim_{t \to 0} e^{-3t} \right) i + \left(\lim_{t \to 0} \frac{t^2}{\sin^2 t} \right) j + \left(\lim_{t \to 0} \cos(2t) \right) k = i + j + k$$

$$= \underline{1 i + j + k}$$

L'hopital's Rule OR

$$\lim_{t \to 0} \frac{1}{\frac{\sin^2 t}{t^2}} = \lim_{t \to 0} \frac{1}{\left(\frac{\sin t}{t}\right)^2} = \frac{1}{1^2} = 1$$

~~1.3~~ 13.1 #3

In general, parametric curves may be more complicated

e.g $\vec{r}(t) = \cos t\, i + \sin t\, j + t\, k$

Parametric equations:

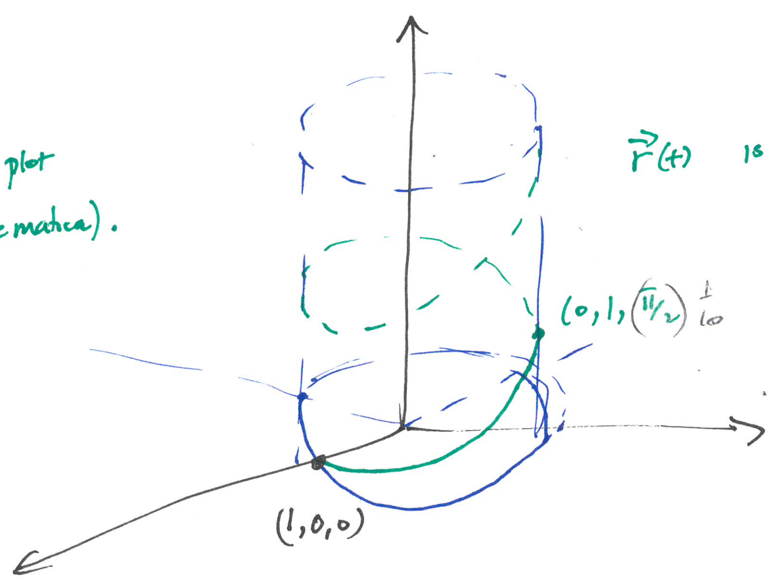
$x = \cos(t)$
$y = \sin(t)$
$z = \frac{t}{10}$.

If we project $\vec{r}(t)$ to the x-y plane i.e set $z=0$
$x = \cos(t)$, $y = \sin(t)$ becomes
$x^2 + y^2 = 1$.

This means that as t increases, the curve ~~x^2+~~ $\vec{r}(t)$ lies in the cylinder $x^2 + y^2 = 1$

Show plot (Mathematica).

$\vec{r}(t)$ is a helix.



Simpler curves

Find the vector equation for the line segment $\overline{PQ}$ [ie joining P and Q]

$P = (2,0,0)$ $Q(6,2,-2)$.

Direction $\vec{v} = \langle 6-2, 2-0, -2\rangle = \langle 4, 2, -2\rangle$.

Parametric equations (assuming the line starts at $P=(2,0,0)$.

$$\vec{r}(t) = (1-t)\vec{r}_0 + t\vec{r}_1. \qquad 0 \le t \le 1$$

$\vec{r}_0 = \langle 2,0,0\rangle$ and $\vec{r}_1 = \langle 6,2,-2\rangle$

$\vec{r}(t) = (1-t)\langle 2,0,0\rangle + t\langle 6,2,-2\rangle$

13.1 #4.

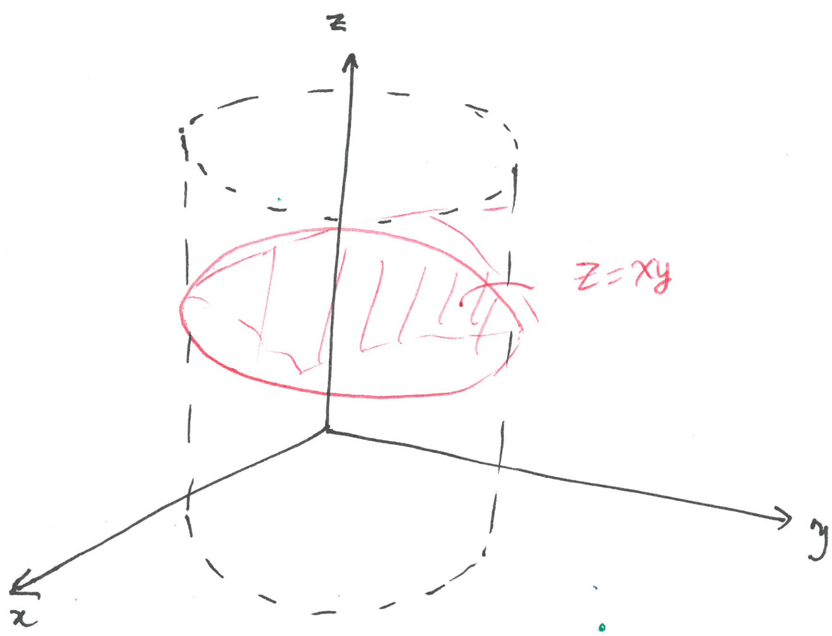
$\vec{r}(t) = \langle 2,0,0\rangle - t\langle 2,0,0\rangle + t\langle 6,2,-2\rangle$

$= \langle 2,0,0\rangle - t\langle 4, 2, -2\rangle$

$$\left.\begin{aligned} x &= 2-4t \\ y &= -2t \\ z &= -2t \end{aligned}\right\} \quad 0 \le t \le 1.$$

Example

Intersection of ~~surface~~ cylinder $x^2+y^2=4$ and surface $z=xy$. [show plot].



$x^2+y^2=4$ Projected onto is a cylinder of radius 2 with center at $(0,0)$

$x = 2\cos t \qquad y = 2\sin t \qquad 0 \le t \le 2\pi$

$z = xy. = (2\cos t)(2\sin t) = 4\cos(t)\sin(t).$

The parametric equation for the intersection is

$\vec{r}(t) = (2\cos t)\,i + (2\sin t)\,j + 4\cos(t)\sin(t)\,k.$