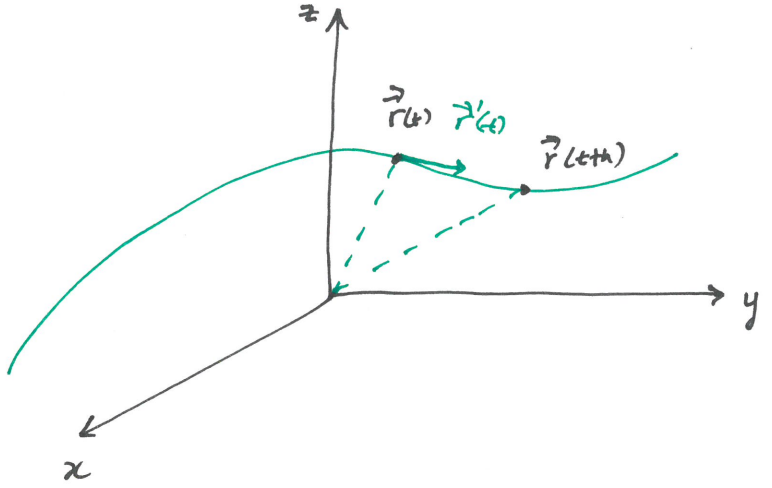
12.4 Motion in space. Suppose a particle moves in space with position vector given by $\vec{r}(t)$.



① Velocity $= \lim_{h\to 0} \dfrac{\vec{r}(t+h) - \vec{r}(t)}{h} = \vec{r}\,'(t)$.

② Speed is the magnitude of the velocity vector

$$|\vec{v}(t)| = |\vec{r}\,'(t)| = \frac{ds}{dt} = \text{Rate of change of distance with respect to time.}$$

③ Acceleration – rate of change of velocity

$$\vec{a}(t) = \vec{v}\,'(t) = r''(t).$$

Example

Find the velocity, acceleration and speed of a particle with position vector

$$\vec{r}(t) = t\,i + t^2 j + 2k, \text{ @ } t=1.$$

① velocity $\vec{v}(t) = \vec{r}\,'(t) = \frac{d}{dt}(t)\,i + \frac{d}{dt}(t^2)\,j + \frac{d}{dt}(2)\,k.$

$= i + (2t)\,j + 0\cdot k$

$= i + (2t)j \quad = \langle 1, 2t, 0\rangle$

$= \langle 1, 2, 0\rangle$ @ $t=1$.

② Speed of particle @ $t=1$ is $\sqrt{1^2+2^2+0^2} = \sqrt{5}$

③ Acceleration $a(t) = \vec{r}\,''(t) = 0i + 2j + 0k \quad = \langle 0, 2, 0\rangle$.

12.4

Find the position vector of a particle with the specified acceleration and specified velocity and position.

$\vec{a}(t) = 2t\,i + \sin t\,j + \cos(2t)\,k$, $\vec{v}(0) = i$, $\vec{r}(0) = j$.

$$\vec{v}(t) = \int 2t\,i + \sin t\,j + \cos(2t)\,k = t^2 i - \cos t\,j + \tfrac{1}{2}\sin 2t\,k + C$$

$$\vec{v}(0) = i \Rightarrow -\cos(0)\,j + \vec{C} = i$$

$$\langle 0, -1, 0\rangle + \vec{C} = \langle 1, 0, 0\rangle$$

$$\vec{C} = \langle 1, 1, 0\rangle = i + j.$$

$$\vec{v}(t) = t^2 i + i - \cos t\,j + j + \tfrac{1}{2}\sin 2t\,k$$

$$= (t^2+1)\,i + (1 - \cos t)\,j + \tfrac{1}{2}\sin 2t\,k.$$

$$\vec{r}(t) = \int \vec{v}(t)\,dt = \int (t^2+1)\,i + (1-\cos t)\,j + \tfrac{1}{2}\sin(2t)\,k$$

$$= \left(\tfrac{t^3}{3} + t\right) i + (t - \sin t)\,j - \tfrac{1}{4}\cos(2t)\,k + \vec{D}$$

$\vec{r}(0) = j \Rightarrow$ solving for $\vec{D}$ $\quad \vec{D} = j + \frac{k}{4}$ (exercise!)

$$\boxed{\vec{r}(t) = \left(\tfrac{t^3}{3} + t\right) i + (t - \sin t + 1)\,j + \left(\tfrac{1}{4} - \tfrac{1}{4}\cos(2t)\right) k}$$