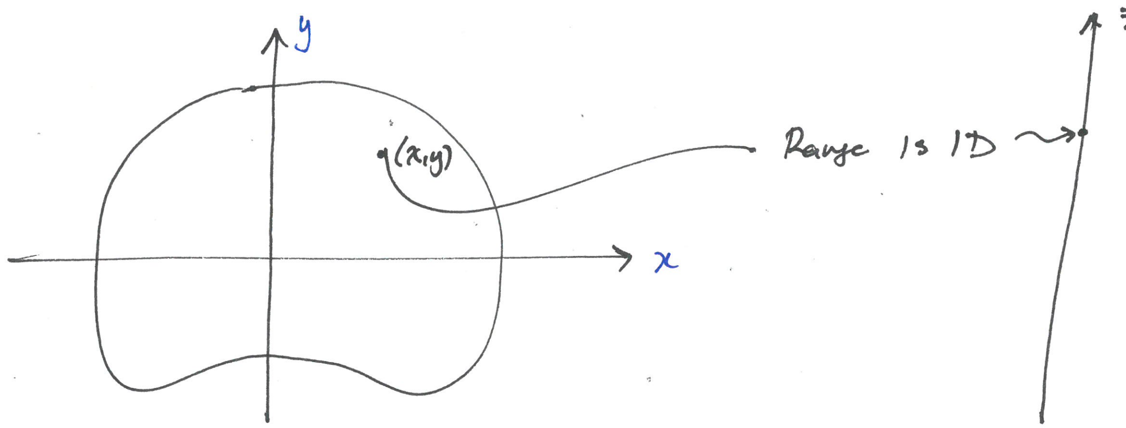
Function of 2 variables

A function of two variables assigns to each (x,y) in the domain D, a unique number $f(x,y)$ in the Range.



Domain is 2 dimensional

Independent variables
$(x,y) \in \mathbb{R}^2$

Dependent variable
$z = f(x,y)$.
$\in \mathbb{R}$.

Example #1

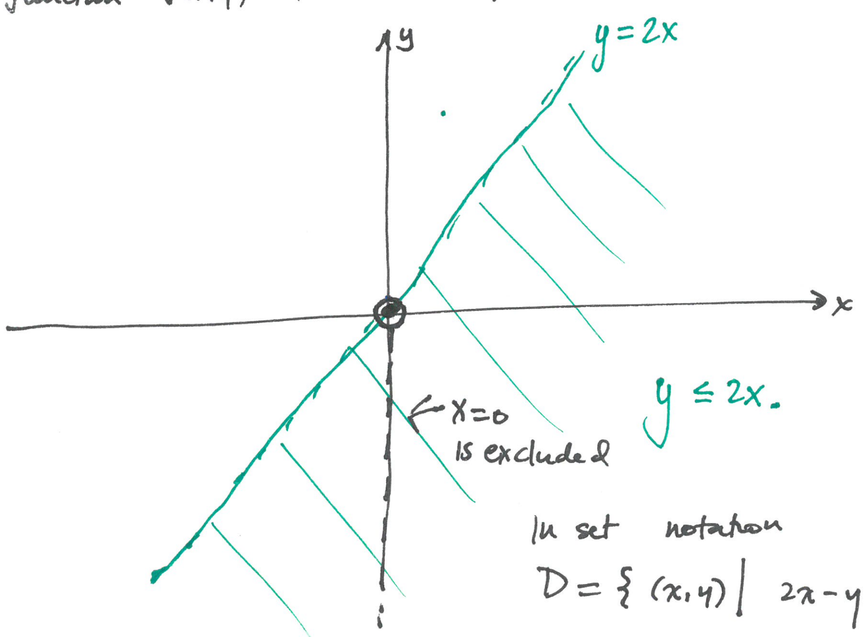
Find and sketch the domain.

(a) $f(x,y) = \dfrac{\sqrt{2x-y}}{x}$

~~Domain is~~
The function $f(x,y)$ is well defined for $2x-y \geq 0$ and $x \neq 0$

$2x - y \geq 0$

$2x \geq y$



In set notation

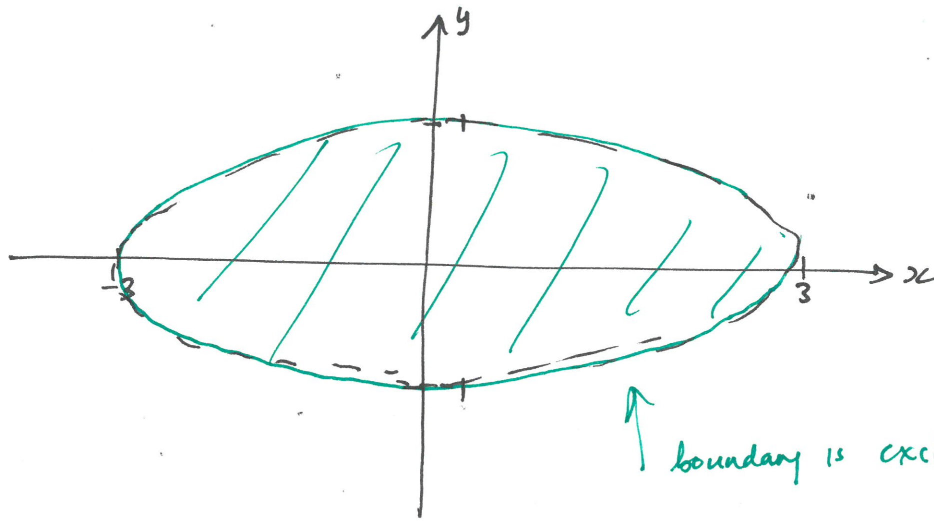
$D = \{ (x,y) \mid 2x - y \geq 0 \text{ and } x \neq 0 \}$.

$f(x,y) = \ln(9 - x^2 - 9y^2)$

$f(x,y)$ is well defined for $9 - x^2 - 9y^2 > 0$

$x^2 + 9y^2 \leq 9$

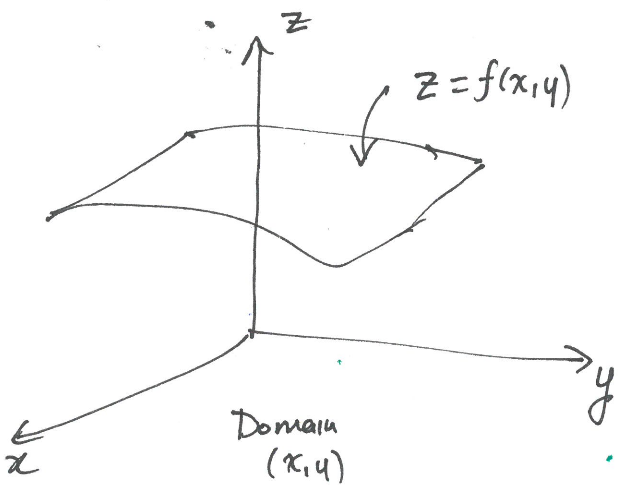
$x^2 + 9y^2 \leq 9$ is the interior an ellipse $D = \{(x,y) \mid x^2 + 9y^2 \leq 9\}$.



boundary is excluded.

Why multi-variable

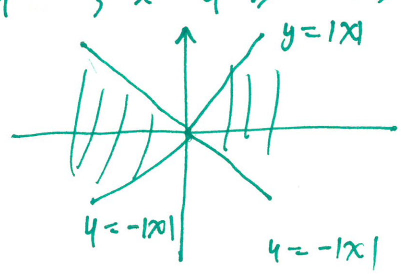
We want to describe functions in 3D space.



Ex, they do (Worksheet)

$f(x) = \sqrt{x^2 - y^2}$, $x^2 - y^2 \geq 0 \Leftrightarrow y^2 \leq x^2 \Leftrightarrow |y| \leq |x| \Rightarrow$

$-|x| \leq |y| \leq |x|$



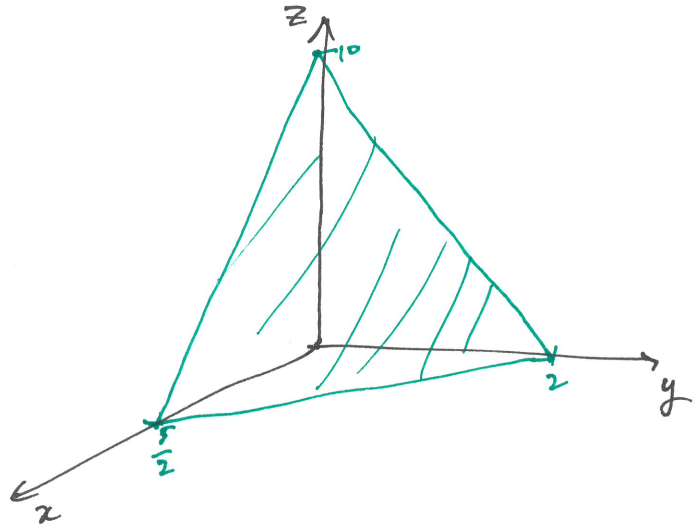
Graph of a 2 variable function

- The set of all points (x, y, z) in $\mathbb{R}^3$ such that $z = f(x,y)$ and $(x,y) \in$ Domain.

<u>Example</u>

<u>Planes</u>

$f(x,y) = \cancel{10}\ 10 - 4x - 5y$



Plot intercepts

@ $y=0$, $z=0$

$0 = 10 - 4x$

$\frac{4x}{4} = \frac{10}{4}$

$x = 2.5$

@ $x=0$, $z=0$

$0 = 10 - 5y$

$y = 2$

@ $x=y=0$

$z = 10$

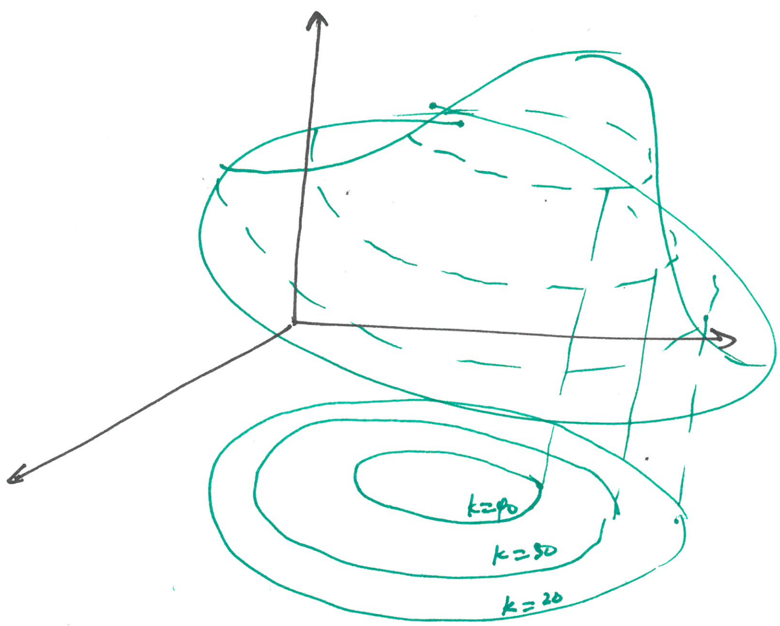
<u>Matlab plots of various functions</u>

Transition to level curves.

Level curves

The level curve of $f(x,y)$ are curves with equation

$$f(x,y) = k$$



* Level curves show where the graph of f has a height of k

Remarks

1. Level curves are like contour maps.
2. Level curves are traces of the graph of f along $z=k$ projected to the ~~xy~~ xy plane.
3. A common example is Topographical maps.

Example

Sketch the level curves of $f(x,y) =$

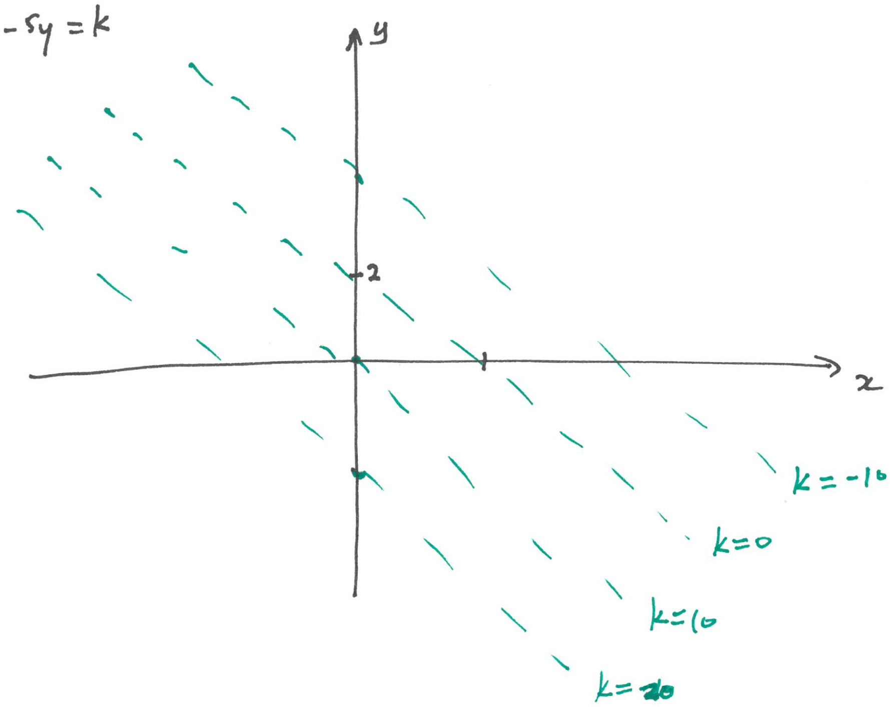
$$f(x,y) = 10 - 4x - 5y.$$

$$f(x,y) = k \Rightarrow 10 - 4x - 5y = k$$

$$10 - 4x - k = 5y \Rightarrow y = -\frac{4}{5}x + \frac{1}{5}(10-k)$$

This is a family of lines of slope $-\frac{4}{5}$ with intercept @ $\frac{1}{5}(10-k)$.

14.1 $10 - 4x - 5y = k$ #5



Example #2

level curves of the function

$$g(x,y) = \sqrt{9 - x^2 - y^2} \quad , \quad k = 0, 1, 2, 3$$

Level curves $g(x,y) = k \Rightarrow \sqrt{9 - x^2 - y^2} = k$

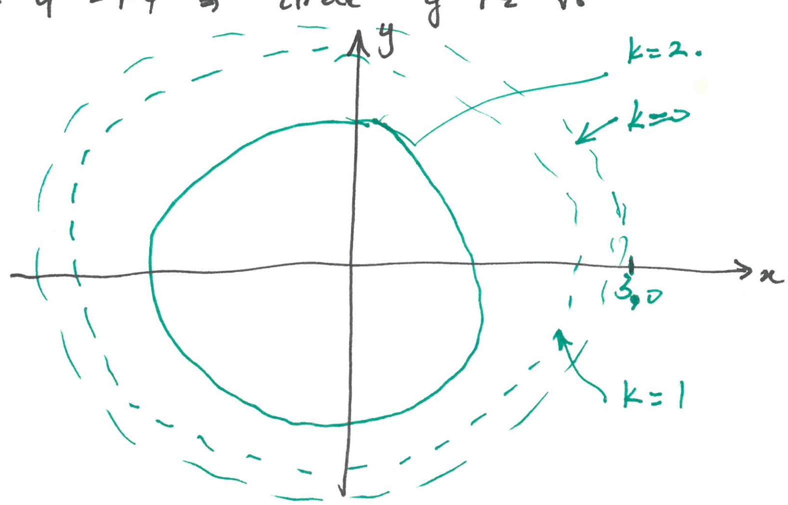
$\Rightarrow 9 - x^2 - y^2 = k^2$

$x^2 + y^2 = 9 - k^2$

$k=0$ - circle of radius 3

$k=1 \Rightarrow x^2 + y^2 = 8 \Rightarrow$ circle of radius $\sqrt{8}$

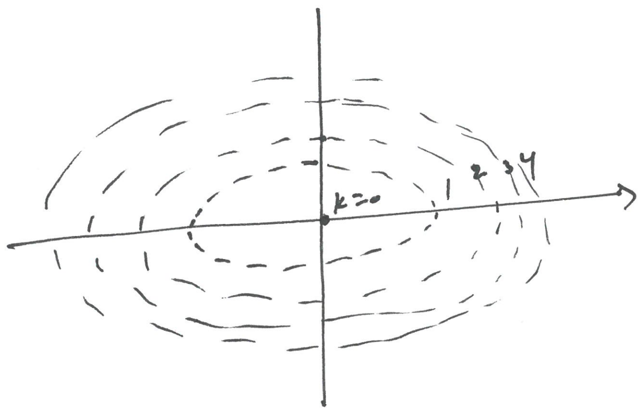
$k=2 \Rightarrow x^2 - y^2 = 9-4 \Rightarrow$ circle of $r = \sqrt{5}$

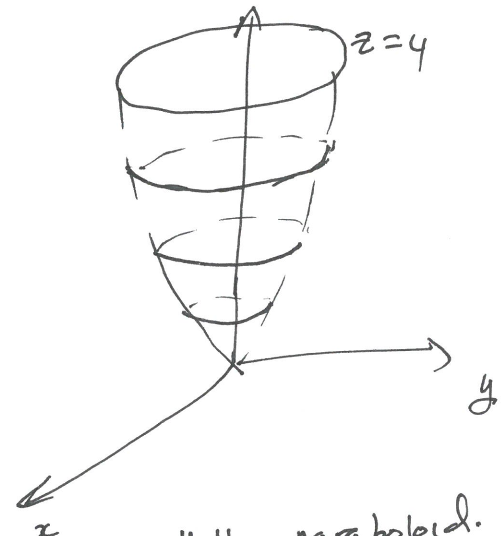


Examples

$f(x,y) = x^2 + 9y^2$

The contour map consist of level surfaces $k = x^2 + 9y^2$.





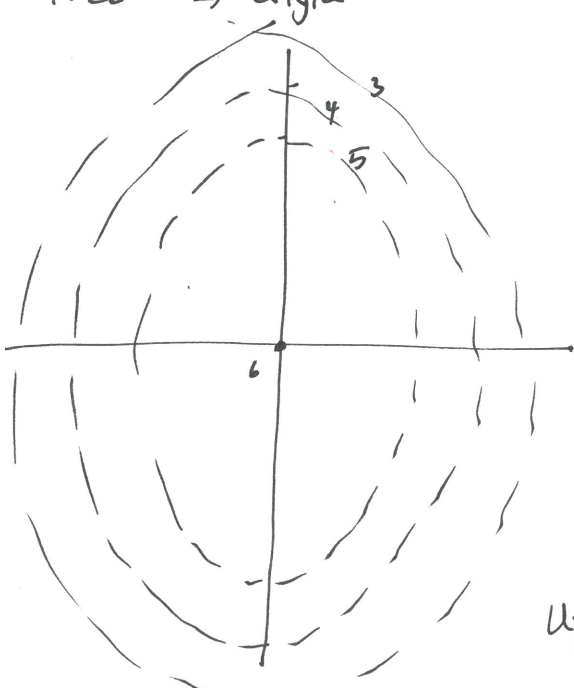
elliptic paraboloid.

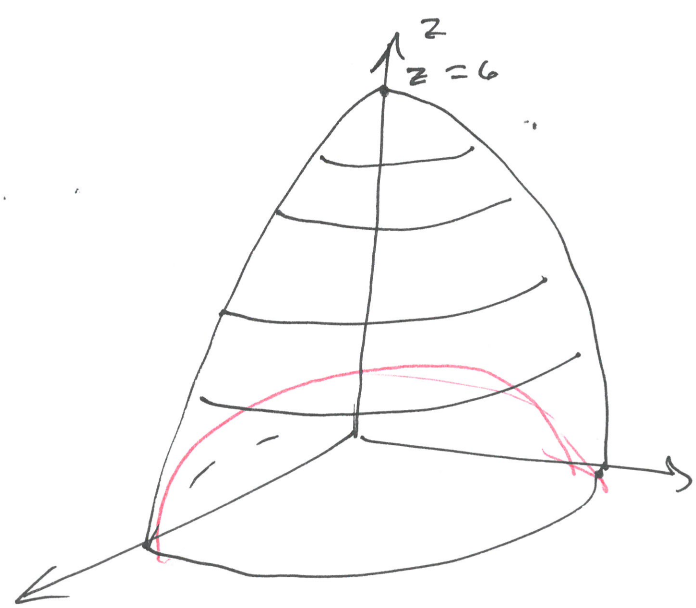
$f(x,y) = \sqrt{36 - 9x^2 - 4y^2}$ [level curves show, where the graph of f has a height of k]

$k = \sqrt{36 - 9x^2 - 4y^2} \Rightarrow 9x^2 + 4y^2 = 36 - k^2,$

$k \geq 0$ is a family of ellipses with major axis the y-axis

$k = 0 \Rightarrow$ origin





Upper half of ellipsoid.

Functions of N-variables

<u>N=3</u>

$q = f(x,y,z)$, assigns to each (x,y,z) in $\mathbb{R}^3$ a unique number $f(x,y,z)$.

Eg ① $T = f(x,y,t)$ - Temperature depends on position (x,y) and t. (time of day)

② $T = f(x,y,h)$ - Temperature depends on (x,y,h) - long, latitude and height.

<u>In general</u>, a function of n variables assigns

$z = f(x_1, x_2, \ldots, x_n)$ to each $(x_1, x_2, \ldots, x_n) \in \mathbb{R}^n$

It is often convinient to write

$z = \vec{f}(\vec{x})$ where

$\vec{x} = \langle x_1, x_2, \ldots, x_n \rangle$.

<u>Examples</u>

(a) $f(x,y,z) = x + 3y + 5z$

$x + 3y + 5z = k$

- Family of parallel planes with normal vector $\vec{n} = \langle 1,3,5 \rangle$.

(b) $f(x,y,z) = y^2 + z^2$

$y^2 + z^2 = k$, $k > 0$, a family of circular cyclinder with the axis x-axis and $r = \sqrt{k}$.