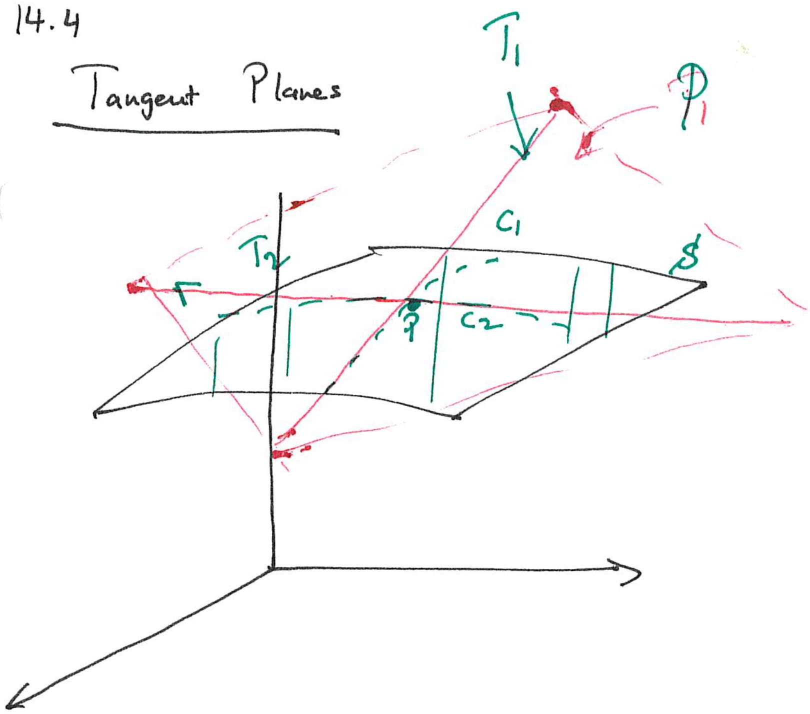
14.4 #1

Tangent Planes



$z = f(x,y)$ with continuous first partial derivatives

$P(x_0, y_0, z_0)$ lives on S

- Let C_1 and C_2 be the intersection of S and the vertical planes $y = y_0$ and $x = x_0$.

let $\vec{n} = \langle A, B, C \rangle$ be the normal vector to the Tangent plane P_1

T_1 has equation $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0.$

We can write

$$C(z - z_0) = -A(x - x_0) - B(y - y_0)$$

$$\boxed{z - z_0 = \frac{-A}{C}(x - x_0) \frac{-B}{C}(y - y_0)} \quad (*)$$

T_1 and T_2 both live on the plane P.

If we plug in $y = y_0$ into $(*)$ we get

$$z - z_0 = -\frac{A}{C}(x - x_0)$$ which is the point-slope form of the line with slope $\frac{-A}{C}$, but recall that the slope of T_1 is $f_x(x_0, y_0)$! , Similarly $-\frac{B}{C} = f_y(x_0, y_0)$

So the equation of the plane is

$$\boxed{z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)}$$

Example

Tangent ~~line~~ plane to $z = \ln(x - 2y)$ at $P = (3, 1, 0)$

$$f_x(x,y) = \frac{1}{x-2y} \cdot 1 = \frac{1}{x-2y} \Rightarrow f_x(3,1) = \frac{1}{3-2\cdot 1} = 1$$

$$f_y(x,y) = \frac{1}{x-2y} \cdot -2 = \frac{-2}{x-2y} \Rightarrow f_y(3,1) = \frac{-2}{3-2\cdot 1} = -2.$$

$$z - 0 = 1(x-3) + (-2)(y-1)$$

$$\boxed{z = (x-3) - 2(y-1)}$$

Linear approximations

"x"

$x - 2y - 1$

In example 2, we found $z = (x - 3) - 2(y - 1)$ is the tangent plane to

$z = \ln(x - 2y)$.

$z = x - 2y - 1$ is a linear function of 2 variables

$$L(x,y) = x - 2y - 1$$

For points near $(3,1)$, we can approximate $f(x,y)$ using the tangent plane

e.g $\quad f(2.95, 0.95) = 2.95 - (2 \cdot 0.95) - 1 = 0.05$

True value

$$y = \ln(x - 2y).$$

$$y = \ln(2.95 - 2 \cdot 0.95)$$

$$= \ln(1.05) = \underline{0.0487}$$

For points further away, the approximation becomes less accurate.

In Summary.

$$f(x,y) \cong f(a,b) + f_x(a,b)(x - a) + f_y(a,b)(y - b)$$

is called the Linearization of f at (a,b) $\quad f(x,y) \cong L(x,y)$ near (a,b).

For 3 variable functions

$$f(x,y,z) \cong f(a,b,c) + f_x(a,b,c)(x-a) + f_y(a,b,c)(y-b) + f_z(a,b,c)(z-c)$$