Name: Solutions

Sec. 2.5 - Continuity

Math 251

1. Explain why the function

$$f(x) = \begin{cases} x+3 & \text{if } x \le -1 \\ 2^x & \text{if } x > -1 \end{cases}$$

is discontinuous at $x = -1$.

$\lim_{x \to -1^-} f(x) = -1+3 = 2$ but $\lim_{x \to -1^+} f(x) = 2^{-1} = \frac{1}{2}$ so f is discontinuous at $x = -1$

2. For what values of the constant c is

$$f(x) = \begin{cases} cx^2 + 2x, & \text{if } x < 2 \\ x^3 - cx, & \text{if } x \ge 2 \end{cases}$$

continuous on $(-\infty, \infty)$.

We need c such that $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$ so

$$c \cdot 2^2 + 2 \cdot 2 = 8 - 2c$$

$$4c + 4 = 8 - 2c$$

$$6c = 4 \Rightarrow c = \frac{2}{3}$$

3. Evaluate $\lim_{x \to 1} \arcsin\left(\frac{1-\sqrt{x}}{1-x}\right)$.

Since arcsin is continuous on its domain

$$\lim_{x \to 1} \arcsin\left(\frac{1-\sqrt{x}}{1-x}\right) = \arcsin\left(\lim_{x \to 1}\left(\frac{1-\sqrt{x}}{1-x}\right)\right)$$

$$= \arcsin\left(\lim_{x \to 1} \frac{1}{1+\sqrt{x}}\right)$$

$$= \arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}.$$