Name:
Sec. 2.6 - Limits at Infinity
Math 251

1. $\lim\limits_{x\to\infty} \dfrac{x^2}{x^4+1}$

$$\lim_{x\to\infty}\left(\frac{\frac{x^2}{x^4}}{\frac{x^4}{x^4}+\frac{1}{x^4}}\right) = \lim_{x\to\infty}\left(\frac{\frac{1}{x^2}}{1+\frac{1}{x^4}}\right) = \frac{0}{1+0} = 0$$

2. $\lim\limits_{x\to\infty} \left[\ln(2+x) - \ln(1+x)\right]$

$$\ln(2+x) - \ln(1+x) = \ln\left(\frac{2+x}{1+x}\right), \text{ Since } \ln(x) \text{ is } \underline{\text{continuous}} \text{ for } x>0$$

$$\lim_{x\to\infty}\left[\ln\left(\frac{2+x}{1+x}\right)\right] = \ln\left(\lim_{x\to\infty}\left(\frac{2+x}{1+x}\right)\right) = \ln\left(\lim_{x\to\infty}\left(\frac{\frac{2}{x}+1}{\frac{1}{x}+1}\right)\right) = \ln(1) = 0$$

3. $\lim\limits_{x\to\infty} (e^{-2x}\cos(x))$

$$-e^{-2x} \le e^{-2x}\cos(x) \le e^{-2x}$$

Since $\lim\limits_{x\to\infty}(-e^{-2x}) = 0 = \lim\limits_{x\to\infty}(e^{-2x})$, by the Squeeze Theorem

$$\lim_{x\to\infty}(e^{-2x}\cos(x)) = 0$$

4. $\lim\limits_{x\to\infty} (e^{-x} + 2\cos(3x))$

The limit does not exist because while $\lim\limits_{x\to\infty}(e^{-x}) = 0$, $\lim\limits_{x\to\infty} 2\cos(3x)$ DNE

because $2\cos(3x)$ oscillates between -2 and 2 as $x \to \infty$.