

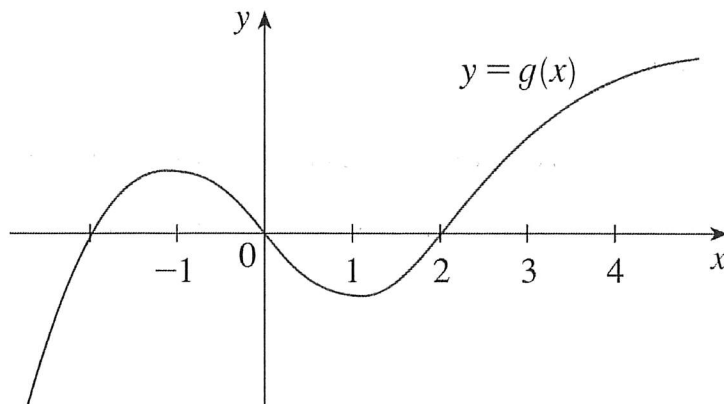
Name:

Sec. 2.7 - Derivatives and Rates of change

Math 251

1. For the function g whose graph is given, arrange the following numbers in increasing order and explain your reasoning:

$$0 \quad g'(-2) \quad g'(0) \quad g'(2) \quad g'(4)$$



$g'(0)$ is the only negative value. The slope at $x=4$ is smaller than at $x=2$ and both are smaller than at $x=-2$. Thus

$$g'(0) < 0 < g'(4) < g'(2) < g'(-2)$$

2. Find $f'(a)$ for $f(x) = x^{-2}$

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \rightarrow 0} \left(\frac{\frac{1}{(a+h)^2} - \frac{1}{a^2}}{h} \right)$$

$$= \lim_{h \rightarrow 0} \left(\frac{-2a-h}{(a+h)^2 a^2} \right)$$

$$= \frac{-2a}{a^2 \cdot a^2} = \frac{-2a}{a^4} = \frac{-2a}{a^3}$$

Simplify

$$\frac{1}{h} \left(\frac{1}{(a+h)^2} - \frac{1}{a^2} \right)$$

$$= \frac{1}{h} \left(\frac{a^2 - (a+h)^2}{(a+h)^2 a^2} \right)$$

$$= \frac{1}{h} \left(\frac{a^2 - (a^2 + 2ah + h^2)}{(a+h)^2 a^2} \right)$$

$$= \frac{1}{h} \left(\frac{-(2ah + h^2)}{(a+h)^2 a^2} \right) = \frac{h}{h} \frac{(-2a - h)}{(a+h)^2 a^2}$$

3. The number of bacteria after t hours in a controlled laboratory experiment is $n = f(t)$.

(a) What is the meaning of $f'(5)$? What are the units?

$f'(5)$ is the rate of growth of bacteria when $t = 5$ hrs. units: bacteria/hour.

(b) Suppose there is an unlimited amount of space and nutrients for the bacteria. Which do you think is larger, $f'(5)$ or $f'(10)$? If the supply of nutrients is limited, would that affect your conclusion? Explain.

With unlimited space and nutrients f' should increase as t increases

so $f'(5) < f'(10)$.

If the supply of nutrients is limited, the growth slows down at some point in time so it may be the case that $f'(5) > f'(10)$.