Name:
Sec. 3.3 - Derivatives of trigonometric functions
Math 251

1. Show that $\dfrac{d}{dx}(\sec(x)) = \sec x \tan x$

Recall $\sec(x) = \dfrac{1}{\cos(x)}$

$$\frac{d}{dx}\left(\frac{1}{\cos(x)}\right) = \frac{\cos(x)\cdot 0 - 1\cdot -\sin(x)}{\cos^2(x)} = \frac{+\sin(x)}{\cos(x)\cdot\cos(x)} = \frac{1}{\cos(x)}\cdot\frac{\sin(x)}{\cos(x)}$$

$$= \sec(x)\tan(x).$$

2. Suppose that $f(\frac{\pi}{3}) = 4$ $f'(\frac{\pi}{3}) = -2$ and let $g(x) = f(x)\sin(x)$ and $h(x) = \dfrac{\cos(x)}{f(x)}$. Find

(a) $g'(\frac{\pi}{3})$

$$g'(x) = f'(x)\sin(x) + f(x)\cos(x)$$

$$g'(\tfrac{\pi}{3}) = f'(\tfrac{\pi}{3})\sin(\tfrac{\pi}{3}) + f(\tfrac{\pi}{3})\cos(\tfrac{\pi}{3})$$

$$= -2\cdot\frac{\sqrt{3}}{2} + 4\cdot\frac{1}{2} = \underline{-\sqrt{3}+2}$$

(b) $h'(\frac{\pi}{3})$

$$h'(x) = \frac{f(x)\cdot -\sin(x) - \cos(x)\cdot f'(x)}{(f(x))^2}$$

$$h'\left(\frac{\pi}{3}\right) = \frac{f(\frac{\pi}{3})\cdot\left(-\sin(\frac{\pi}{3})\right) - \cos(\frac{\pi}{3})f'(\frac{\pi}{3})}{4^2}$$

$$= \frac{4\cdot\left(-\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\cdot(-2)}{16} = \frac{-2\sqrt{3}+1}{16}$$