Name:
Sec. 3.3 - Derivatives of trigonometric functions
Math 251

1. Show that \(\frac{d}{dx}(\sec(x)) = \sec x \tan x\)

Recall \(\sec(x) = \frac{1}{\cos(x)}\)

\[
\frac{d}{dx}\left(\frac{1}{\cos(x)}\right) = \frac{\cos(x) \cdot 0 - 1 \cdot -\sin(x)}{\cos^2(x)} = \frac{\sin(x)}{\cos(x) \cdot \cos(x)} = \frac{1}{\sec(x)} \cdot \sin(x) \cos(x) = \sec(x) \tan(x).
\]

2. Suppose that \(f(\frac{\pi}{3}) = 4\) \(f'(\frac{\pi}{3}) = -2\) and let \(g(x) = f(x) \sin(x)\) and \(h(x) = \frac{\cos(x)}{f(x)}\). Find

(a) \(g'(\frac{\pi}{3})\)

\[
g'(x) = f'(x) \sin(x) + f(x) \cos(x)
\]

\[
g'(\frac{\pi}{3}) = f'(\frac{\pi}{3}) \sin\left(\frac{\pi}{3}\right) + f\left(\frac{\pi}{3}\right) \cos\left(\frac{\pi}{3}\right)
\]

\[
= -2 \cdot \frac{\sqrt{3}}{2} + 4 \cdot \frac{1}{2} = -\sqrt{3} + 2
\]

(b) \(h'(\frac{\pi}{3})\)

\[
h'(x) = \frac{f(x) \cdot \frac{\sin(x)}{x} - \cos(x) \cdot f'(x)}{(f(x))^2}
\]

\[
h'(\frac{\pi}{3}) = \frac{f\left(\frac{\pi}{3}\right) \cdot \left(-\sin\left(\frac{\pi}{3}\right)\right) - \cos\left(\frac{\pi}{3}\right) f'(\frac{\pi}{3})}{4^2}
\]

\[
= \frac{4 \cdot \left(-\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right) \cdot (-2)}{16} = \frac{-2\sqrt{3} + 1}{16}
\]