Name:
Sec. 3.5 - Implicit Differentiation
Math 251

1. Find $\frac{dy}{dx}$ by implicit differentiation

(a) $x^2 - 4xy + y^2 = 4$

$$2x - 4x \cdot \frac{dy}{dx} + y \cdot 4 + 2y \cdot \frac{dy}{dx} = 0$$

$$(2y - 4x)\frac{dy}{dx} = -(2x + 4y) \Rightarrow \frac{dy}{dx} = \frac{-(2x+4y)}{2y - 4x}.$$

(b) $\cos(xy) = 1 + \sin(y)$

$$-\sin(xy)\frac{d}{dx}(xy) = 0 + \cos(y)\frac{dy}{dx}$$

$$-\sin(xy)\left(x \cdot \frac{dy}{dx} + y \cdot 1\right) = \cos(y)\frac{dy}{dx}$$

$$-x\sin(xy)\frac{dy}{dx} - y\sin(xy) = \cos(y)\frac{dy}{dx}$$

$$-y\sin(xy) = \left[\cos(y) + x\sin(xy)\right]\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{-y\sin(xy)}{\cos(y) + x\sin(xy)}$$

(c) $e^{\frac{x}{y}} = x - y$

$$e^{\frac{x}{y}} \cdot \frac{d}{dx}\left(\frac{x}{y}\right) = 1 - \frac{dy}{dx}$$

$$e^{\frac{x}{y}} \cdot \left[\frac{y \cdot 1 - x \cdot y'}{y^2}\right] = 1 - y'$$

$$\frac{e^{\frac{x}{y}} \cdot y}{y^2} - \frac{e^{\frac{x}{y}} \cdot xy'}{y^2} = 1 - y'$$

$$\frac{e^{\frac{x}{y}}}{y} - 1 = \left(1 + \frac{e^{\frac{x}{y}} \cdot x}{y^2}\right) y'$$

$$y' = \frac{\frac{e^{\frac{x}{y}}}{y} - 1}{1 + \frac{e^{\frac{x}{y}} \cdot x}{y^2}}$$

(d) $\tan^{-1}(x^2)$

$$\frac{1}{1 + (x^2)^2} \cdot 2x$$