Name:
Sec. 3.6 - Logarithmic functions
Math 251
Find the derivatives of the following functions

1. $y = \ln(e^{-x} + xe^{-x})$

Chain Rule

$$\frac{dy}{dx} = \frac{1}{e^{-x} + xe^{-x}} \cdot \frac{d}{dx}\left(e^{-x} + xe^{-x}\right)$$

(Product!)

$$= \frac{1}{e^{-x} + xe^{-x}} \cdot \left[-e^{-x} + x(-e^{-x}) + e^{-x} \cdot 1\right]$$

2. $x^y = y^x$

$$\ln(x^y) = \ln(y^x)$$

$$y\ln(x) = x\ln(y)$$

$$y' \cdot \ln(x) + y \cdot \frac{1}{x} = x \cdot \frac{1}{y} y' + \ln(y) \cdot 1$$

$$y' \cdot \ln(x) - \frac{x}{y} y' = \ln(y) - \frac{y}{x}$$

$$y'\left(\ln(x) - \frac{x}{y}\right) = \ln(y) - \frac{y}{x}$$

$$\boxed{y' = \left(\frac{\ln(y) - \frac{y}{x}}{\ln(x) - \frac{x}{y}}\right)}$$

3. $y = \ln(x^2 + y^2)$

$$y' = \frac{1}{x^2+y^2} \cdot \frac{d}{dx}(x^2+y^2)$$

$$= \frac{1}{x^2+y^2} \cdot \left[2x + 2y \cdot \frac{dy}{dx}\right]$$

$$y' = \frac{2x}{x^2+y^2} + \frac{2yy'}{x^2+y^2}$$

$$y'\left[1 - \frac{2y}{x^2+y^2}\right] = \frac{2x}{x^2+y^2}$$

$$y'\left[\frac{x^2+y^2-2y}{x^2+y^2}\right] = \frac{2x}{x^2+y^2}$$

$$\boxed{y' = \frac{2x}{x^2+y^2-2y}}$$

4. $y = (\ln(x))^{\cos(x)}$

$$\ln(y) = \ln\left(\ln(x)^{\cos(x)}\right)$$

$$= \cos(x)\ln(\ln(x))$$

$$\frac{1}{y} y' = \cos(x) \cdot \frac{1}{\ln(x)} \cdot \frac{1}{x} + \ln(\ln(x)) \cdot (-\sin(x))$$

$$y' = y\left[\frac{\cos(x)}{x\ln(x)} + \ln(\ln(x))(-\sin(x))\right].$$