Name:
Sec. 4.2 – Mean Value Theorem
Math 251

1. Show that for $f(x) = 1 - x^{2/3}$ we have $f(-1) = f(1)$ but there is no c such that $f'(c) = 0$. Why does this not contradict Rolle's Theorem?

$f(-1) = f(1) = 0$, However, $f'(x) = -\frac{2}{3}x^{-1/3}$ therefore $f(x)$ is not differentiable at $x = 0$ so Rolle's Theorem does not apply in this case.

2. Show that $f(x) = 2x + \cos x$ has exactly one root on $(-1, 1)$

① We show that $f(x)$ has a root on $(-1,1)$

$\Rightarrow$ Check that $f(x)$ changes sign on $(-1,1)$

$f(-1) = -2 + \cos(-1) < 0$ because $|\cos(x)| < 1$

$f(1) = 2 + \cos(1) > 0$ because $|\cos(x)| < 1$

② To show that $f(x)$ has exactly one root

Compute $f'(x) = 2 + (-\sin(x)) = 2 - \sin(x)$ and

observe that $f'(x) > 1$ for all x. This means that f cannot have 2 roots because ~~it~~ this would require (by Rolle's Thm) the existence of a point c in $(-1,1)$ s.t.

$f'(c) = 0$, but c does not exist because $f'(x) > 1$.