1. A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. There is no need for fencing along the river. What are the dimensions of the field that has the largest area?

2. A company wants to design a cylindrical can to hold 1 L of oil. Find the dimensions that will minimize the cost of the metal to manufacture the can.

3. What is the minimum vertical distance between the parabolas \( y = x^2 + 1 \) and \( y = x - x^2 \)?

4. A retailer sells tablets. Suppose the price/demand function is \( p(x) = -\frac{1}{8}x + 500 \) for \( x \geq 1200 \).
   
   (a) What should the price be set at in order to maximize revenue?
   
   (b) If the retailer’s cost function is
   
   \[ C(x) = 35,000 + 120x \]
   
   what price should it choose in order to maximize its profit?

5. Find the dimensions of the largest rectangle that can be inscribed in an ellipse \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \).

For Problems 1, 2, 3, and 5 see class notes.

4(b)

\[ \Pi(x) = R(x) - C(x) \]

Profit = Revenue - Cost

so

\[ \Pi(x) = x \cdot p(x) - C(x) \]

\[ = x \cdot \left( -\frac{1}{8}x + 500 \right) - \left( 35,000 + 120x \right) \]

\[ = -\frac{x^2}{8} + 500x - 35,000 - 120x \]

\[ = -\frac{x^2}{8} + 350x - 35,000 \]

\[ \Pi'(x) = -\frac{x}{4} + 350 = 0 \quad \Rightarrow \quad x = 1520 \]

Check \( \Pi''(x) = -\frac{1}{4} \) so this is a max.

The price should be \( p(1520) = \$310 \) to maximize profit.