Name:
Sec. 4.7 – Optimization
Math 251

1. A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. There is no need for fencing along the river. What are the dimensions of the field that has the largest area?
2. A company wants to design a cylindrical can to hold 1 L of oil. Find the dimensions that will minimize the cost of the metal to manufacture the can.
3. What is the minimum vertical distance between the parabolas $y = x^2 + 1$ and $y = x - x^2$?
4. A retailer sells tablets. Suppose the price/demand function is $p(x) = -\frac{1}{8}x + 500$ for $x \geq 1200$.
   (a) What should the price be set at in order to maximize revenue?
   (b) If the retailer's cost function is
   $$C(x) = 35,000 + 120x$$
   what price should it choose in order to maximize its profit?
5. Find the dimensions of the largest rectangle that can be inscribed in an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

For Problems 1, 2, 3, 4a, and 5 see class notes!

4(b)

$\Pi(x) = R(x) - C(x)$

PROFIT = REVENUE − COST

Π stands for PROFIT in econ

so

$\Pi(x) = x \cdot p(x) - C(x)$

$= x\cdot\left(-\frac{1}{8}x + 500\right) - \left(35,000 + 120x\right)$

$= -\frac{x^2}{8} + 500x - 35,000 - 120x = -\frac{x^2}{8} + 380x - 35,000$

$\Pi'(x) = -\frac{x}{4} + 380 = 0 \Rightarrow x = 1520$

Check $\Pi''(x) = -\frac{1}{4}$ so this is a max!

The price should be $p(1520) = \$310$ to maximize profit