Name:
Sec. 4.9 – Antiderivatives
Math 251

1. Find $f(x)$ if $f''(x) = 20x^3 - 12x^2 + 6x$ given that $f'(1) = 1$ and $f(1) = 0$

$$f'(x) = \frac{20x^4}{4} - 12\left(\frac{x^3}{3}\right) + 6\left(\frac{x^2}{2}\right) + C = 5x^4 - 4x^3 + 3x^2 + C$$

we want $f'(1) = 1$ so

$5 - 4 + 3 + C = 1 \Rightarrow C = -3$

so $f'(x) = 5x^4 - 4x^3 + 3x^2 - 3$

$$f(x) = 5\left(\frac{x^5}{5}\right) - 4\left(\frac{x^4}{4}\right) + 3\left(\frac{x^3}{3}\right) - 3x + D$$

$$= x^5 - x^4 + x^3 - 3x + D$$

we want $f(1) = 0 \Rightarrow 1 - 1 + 1 - 3 + D = 0 \Rightarrow D = 2$

so $f(x) = x^5 - x^4 + x^3 - 3x + 2$.

2. A particle moves with acceleration $a(t) = 3\cos t - 2\sin t$ and $s(0) = 0$, $v(0) = 4$. Find the displacement function.

$a(t) = v'(t) = 3\cos t - 2\sin t \Rightarrow v(t) = 3\sin t + 2\cos t + C$

$v(0) = 4 \Rightarrow C = 2$ so

$v(t) = 3\sin t + 2\cos t + 2$

$s(t) = -3\cos t + 2\sin t + 2t + D$

$s(0) = 0 \Rightarrow D = 3$

so $s(t) = -3\cos t + 2\sin t + 2t + 3$.