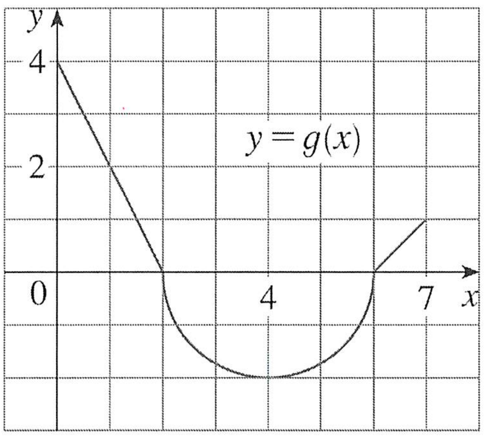
Name:
Sec. 5.2 – The definite integral
Math 251

1. The graph of g is given below Use the graph to evaluate



(a) $\int_0^2 g(x)\,dx$

is the area under the triangle

so $\int_0^2 g(x)\,dx = \frac{1}{2}\cdot 2\cdot 4 = \boxed{4}$

(b) $\int_2^6 g(x)\,dx$

$-\frac{1}{2}\pi\cdot 2^2 = -2\pi$

below the x-axis

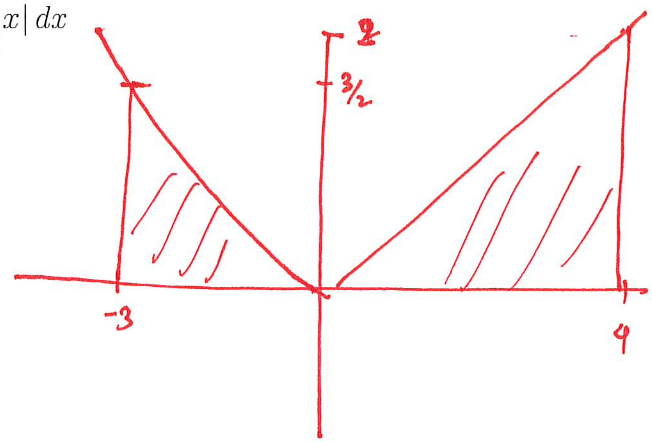
(c) $\int_0^7 g(x)\,dx$

$= \int_0^2 g(x)\,dx + \int_2^6 g(x)\,dx + \int_6^7 g(x)\,dx$

$4 \quad -2\pi \quad + \frac{1}{2}\cdot 1\cdot 1$

2. Evaluate the following

(a) $\int_{-3}^{4} |\frac{1}{2}x|\, dx$



$$\int_{-3}^{4} |\tfrac{1}{2}x|\,dx$$

$$= \frac{1}{2}\cdot 3\cdot\frac{3}{2} + \frac{1}{2}\cdot 4\cdot 2$$

$$= \frac{9}{4} + 4$$

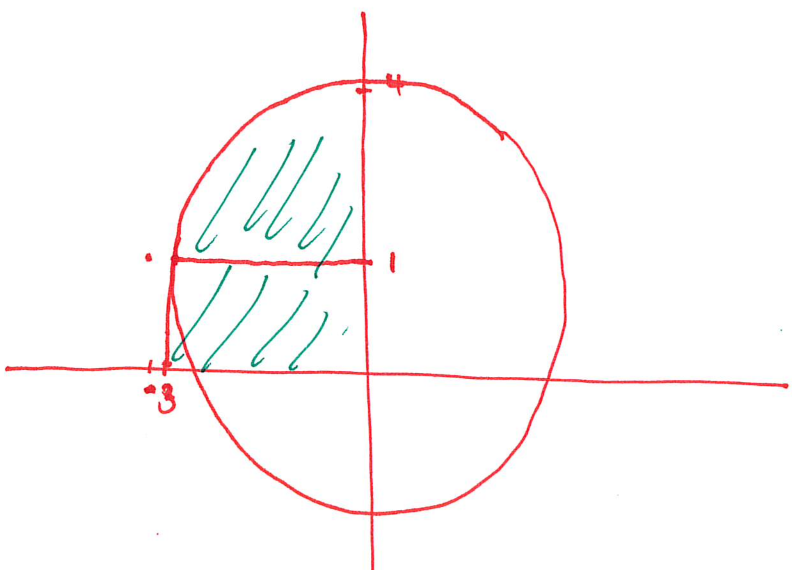
(b) $\int_{-3}^{0} (1+\sqrt{9-x^2})\, dx$

First notice that

$$y = 1+\sqrt{9-x^2} \iff (y-1) = \sqrt{9-x^2}$$

$$\Rightarrow (y-1)^2 = 9-x^2 \Rightarrow x^2+(y-1)^2 = 9$$

is the equation of a circle of $r = \sqrt{9} = 3$ centre $(0,1)$



$$\int_{-3}^{0} (1+\sqrt{9-x^2})\,dx = \frac{1}{2}\cdot 3\cdot 1 + \frac{1}{4}\pi(3)^2$$