1. Verify that

\[ L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix} \quad \text{and} \quad U_1 = \begin{bmatrix} 2 & 7 & 5 \\ 0 & -1 & -5 \\ 0 & 0 & 45 \end{bmatrix} \]

forms the LU decomposition of \( A = \begin{bmatrix} 2 & 7 & 5 \\ 6 & 20 & 10 \\ 4 & 3 & 0 \end{bmatrix} \)

and use the decomposition to solve \( A \alpha = \begin{bmatrix} 5 \\ 4 \\ 1 \end{bmatrix} \)

2. Show that the matrix \( \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \) has no LU decomposition.

3. Let \( A, B \) be \( n \times n \) matrices and \( x \) be a non-zero real number

Show that \( x(A+B) = x(A) + x(B) \)

4. Solve the following system of equations:

\[ \begin{align*}
    x_1 + x_2 &= 2 \\
    x_1 - x_2 &= 0 \\
    2x_1 - x_2 &= 0 
\end{align*} \]

5. Given the data

<table>
<thead>
<tr>
<th>x</th>
<th>-3</th>
<th>1</th>
<th>2</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-23</td>
<td>-11</td>
<td>-23</td>
<td>1</td>
</tr>
</tbody>
</table>

(a) Find the Newton form of the interpolating polynomial

(b) Show that \( f(x) = x^2 - 3x^2 - 16x + 1 \) also interpolates the data

(c) Why does this not contradict the uniqueness part of the theorem?
Consider \( f(x) = \ln(x) \)

(a) Construct the Lagrange form of the interpolating polynomial for \( f \) passing through \((1, \ln(1)), (2, \ln(2)), (\ln(3))\)

(b) Use your polynomial to estimate \( \ln(1.5) \) and \( \ln(2.4) \)

(c) Establish a theoretical bound for the error using your polynomial to approximate \( \ln(1.5) \).

(d) Compare the theoretical bound to the error i.e. \( \left( \ln(1.5) - P_n(1.5) \right) \)

6. Derive a formula for the number of subintervals on \( [a, b] \) that would be needed to approximate \( f(x) \) so that the error is less than \( \delta \). You may assume that \( \max |f''(x)| < M \).