1. Let \( x_0, x_1, x_2, \ldots, x_n \) be distinct points at which the function \( f \) and its first derivative are defined.

(a) Give a definition of the Hermite interpolant of \( f \).

(b) Suppose \( \varphi_i(x) \) is the Lagrange polynomial of degree \( n \) defined on the data points and we define

\[
H_i(x) = [1 - 2\varphi_i'(x_i)(x - x_i)]\varphi_i^2(x) \\
\hat{H}_i(x) = (x - x_i)\varphi_i^2(x)
\]

i. Show that \( H_i(x) \) and \( \hat{H}_i(x) \) are both polynomials of degree \( 2n + 1 \).

ii. Prove the following properties of \( H_i(x) \) and \( \hat{H}_i(x) \)

\[
H_i(x_j) = \begin{cases} 
1, & \text{for } i = j \\
0, & \text{otherwise}
\end{cases}, \quad \hat{H}_i(x_j) = 0
\]

\[
H'(x_j) = 0, \quad \hat{H}'_i(x_j) = \begin{cases} 
1, & \text{for } i = j \\
0, & \text{otherwise}
\end{cases}
\]

iii. Prove that \( P(x) = \sum_{i=0}^{n} H_i(x)f(x_i) + \sum_{i=0}^{n} \hat{H}_i(x_j)f'(x_i) \) is a Hermite interpolating polynomial of \( f \) of degree \( 2n + 1 \).

2. Problems 9 – 13 on pages 209 – 210