Project 1 : Due on Monday, October 29

Instructions

- You may use codes from the homework, class notes or the text
- Unlike homework assignments, you may not talk to your classmates about your solutions. You can only talk to me about your ideas.
- If you use any root finding algorithms pick a tolerance of $1.0e-07$.
- Your project report should contain any calculations and or justifications of algorithms used for each problem. In addition you should provide all codes, scripts and summary file containing the outputs of all scripts or functions and how you used the output to answer the questions. There is a 5% bonus for project reports typeset in LaTeX.

Problem 1 - Saving for a down payment

A couple plans to open a money market account in which they will save the down payment for the purchase of a home. They have $13,500 from the sale of some stock with which to open the account. After examining their budget, they feel they can deposit an extra $250 into the account each month. What is the minimum interest rate, compounded on a monthly basis, that the couple must earn on their investment to reach their goal of accumulating $25,000 within three years.
Hint: The desired interest is somewhere between 1% *and* 10%

Problem 2 - Volume of chlorine gas

The ideal gas law:

$$PV = nRT$$

relates the pressure (P), volume (V), and temperature (T) of an ideal gas. Here, n represents the number of moles of gas present, and R is the universal gas constant. Real gases satisfy this equation only approximately; under conditions of high pressure and/ or low volume the approximation becomes more crude. One attempt to model the relationship among pressure, volume and temperature for real gases is the van der Waals equation:

$$(P + \frac{n^2 a}{V^2})(V - nb) = nRT,$$

The term involving the parameter a corrects the pressure for intramolecular attractive forces (i.e, the pressure would be higher if not for the attractive forces among the molecules of the gas). The term involving the parameter b is a correction for that proportion of the volume of the gas that is not compressible due to the intrinsic volume of gas molecules. Suppose that one mole of chlorine gas has a pressure of 2 atmospheres and a temperature of 313K. For chlorine gas $a = 6.29 atm \cdot liter^2/mole^2$ and $b = 0.0562 liter/mole$ and the universal gas constant $R = 0.08206 atm \cdot liter/mole \cdot K$. What is the volume of chlorine gas?

Problem 3 - Numerical Differentiation

For $f(x) = \sin(x)$ and $x_0 = \frac{\pi}{3}$ compare the performance of each of the following approximations of the derivatives

A1. $f'(x) \approx \dfrac{f(x+h) - f(x)}{h}$

A2. $f'(x) \approx \dfrac{f(x+h) - f(x-h)}{2h}$

A3. $f'(x) \approx \dfrac{\text{Im}\{f(x+ih)\}}{h}$

1. Implement each method and approximate the error for values of $h = 10^{-i}, i = 0, \ldots, 16$.
2. Plot the $\log_{10}$ of the error as a function of $\log_{10}(h)$ for each method on the same axis in reverse order i.e. starting with large values of h first. You can do this by calling

   ```
   set ( gca, 'xdir', 'reverse' )
   ```

   after your call to the plot function. Comment on your results.
3. Use Taylor's theorem to explain the difference in performance between approximations A1. and A2.
4. Comparing A1-A3, method provides the best approximation for small values of h? Explain why this is the case.

Submission of computational portions

Email your zipped m files named `lastname_project.zip`, including your summary file with a discussion of your results to `pchidyagwai@loyola.edu` with subject `MA427_project`.