**Chapter 3 Solutions**
MATH 251, CALCULUS I, FALL 2018

## Section 3.5

9. $y' = \dfrac{x(x+2y)}{2x^2y + 4xy^2 + 2y^3 + x^2}$

10. $y' = \dfrac{1-e^y}{xe^y + 1}$

14. $y' = \dfrac{1 + y - e^y \cos x}{e^y \sin x - x}$

17. $y' = \dfrac{1 + y^2 - \dfrac{2xy}{1+x^4y^2}}{\dfrac{x^2}{1+x^4y^2} - 2xy}$

20. $y' = \dfrac{(1+x^2)\sec^2(x-y) + 2x\tan(x-y)}{1 + (1+x^2)\sec^2(x-y)}$

25. $y' = \dfrac{-2y\cos(2x) + \cos(2y)}{\sin(2x) + 2x\sin(2y)}$. When $x = \frac{\pi}{2}$ and $y = \frac{\pi}{4}$ $y' = \frac{1}{2}$ so the equation of the tangent line is $y = \frac{1}{2}x$.

27. $y' = \dfrac{2x-y}{x+2y} \Longrightarrow y' = \dfrac{3}{4}$ so the equation of the tangent line is $y - 1 = \frac{3}{4}(x-2)$.

39. $y'' = \frac{1}{e^2}$

51. $y' = \dfrac{1}{\sqrt{1-(2x+1)^2}} \cdot 2$

57. $y' = x \cdot \dfrac{1}{\sqrt{1-x^2}} + \arcsin(x)(1) + \dfrac{1}{2}(1-x^2)^{-1/2}(-2x)$

74a. Recall that the normal line is perpendicular to the tangent line so if the slope of the tangent line is $m$, the slope of the normal line is $\frac{-1}{m}$. $y' = \dfrac{y-2x}{2y-x}$ Slope of tangent at $(-1,1)$ is 1, so the normal line has a slope of $-1$ and its equation is $y = -x$. We can plug this into the equation of the ellipse to obtain the intersection point $x = 1$.

76. Using implicit we can show that $y' = -\frac{x}{4y}$. Let $(a,b)$ be the point on $x^2+4y^2 = 36$ whose tangent line passes through $(12,3)$. The tangent line has equation $y - 3 = -\frac{a}{4b}(x-12)$ (Notice that here I plugged in the point $(a,b)$ into $y'$ to obtain the slope). Since the tangent line passes through $(a,b)$ we have

$$b - 3 = -\frac{a}{4b}(a-12)$$

We have two unknowns so we need another equation to solve for $a$ and $b$. Here we can recall that the point $(a, b)$ lives on the ellipse therefore

$$4b^2 + a^2 = 36$$

Nowe we have two equations and 2 unknowns, solve for $a$ and $b$. You should get $a = 0, b = 3$ or $a = \frac{24}{5}, b = -\frac{9}{5}$

80. This problem is similar to 76. $y' = -\frac{x}{4y}$. Let $h$ be the height of the lamp and let $(a, b)$ be the point of tangency of the line passing through $(3, h)$ and $(-5, 0)$. This line has slope $\frac{1}{8}h$. Now find the slope of the line using the two points $(-50)$ and $(a, b)$ to get $\frac{b}{a+5}$ , then recall that the same slope can also be written interms of the derivative, $-\dfrac{a}{4b}$! This will yeild and equation in $(a, b)$ but then also recall that $(a, b)$ lives on the ellipse therefore $a^2 + 4b^2 = 5$ then solve those 2 equations to get $a = -1$ and $b = 1$ then since $\frac{h}{8} = \frac{b}{a+5}$ we get $h = 2$.

## Section 3.6

2. $f'(x) = \ln x$

4. $f'(x) = 2 \cot x$

9. $g'(x) = \frac{1}{x} - 2$.

16. $y' = \dfrac{1 - 3t^2}{1 + t - t^3}$

26. $y'' = \dfrac{2 + \ln x}{x^2(1 + \ln x)^2}$

31. $f'(1) = 2$

39. $y = 1/e$.

45. $y' = x^{\sin x}\left(\dfrac{sinx}{x} + \ln x \cos x\right)$

46. $y' = \dfrac{1}{2}\sqrt{x}^x(1 + \ln x)$

48. $y' = (\sin x)^{\ln x}\left(\ln x \cot x + \ln \sin x x\right)$

## Section 3.7

5. (a) velocity is positive on $(0, 2)$ and negative on $(2, 3)$. Accelaration is positive on $(0, 1)$ and negative on $(1, 3)$. The particle is speeding up when $v$ and $a$ have the same sign, i.e on $(0, 1)$ when $v > 0$ and $a > 0$. On $(2, 3)$ the particle is slowing down.
(b) $v > 0$ on $(0, 3)$ and $v < 0$ on $(3, 4)$. $a > 0$ on $(1, 2)$ and $a < 0$ on $(0, 1)$ and $(2, 4)$.

9. (a) $v(2) = 7.56m/s$. (b) $t = t_1 \approx 2.35$ or $t = t_2 \approx 5.71$. Evaluating $v(t_1) \approx 6.24m/s$ (upward) and $v(t_2) = -6.24m/s$ (downward)

15. (a) $S'(1) = 8\pi ft^2/ft$ (b) $S'(2) = 16\pi ft^2/ft$ (c) $S'(3) = 24\pi ft^2/ft$.

16. (a) (i) $172\pi\mu m^3/\mu m$ (a)(ii) $121.3\mu m^3/\mu m$ a(iii) $102.013\mu m^3/\mu m$. (b) $V'(r) = \frac{4}{3}\pi r^2$ Evaluate!

17. $\rho(x) = f'(x) = 6x$.

18. $V'(t) = -250(1 - \frac{1}{40}t)$. The water is flowing at its fastest at $t = 0$.

26. $b = 6$ $a = 140$. $\lim_{t\to\infty} \dfrac{140}{1 + 6e^{-0.7t}} = 140$ so the population stabilizes at 140.

31. (a) $C'(x) - 3 + 0.03x + 0.0006x^2$ (b) $C'(100) = \$11/pair.C'(100)$ is the rate at which the cost is increasing as the 100th pair of jeans is being produced. It predicts the approximate cost of the 101st pair. (c) The cost of manufacturing the 101st pair is \$11.07.

32. (a) $C'(q) = 0.16 - 0.0012q + 0.000009q^2$ and $C'(100) = 0.13$. This is the rate at which costs are increasing at the 100th item is produced. (b) The actual cost of producing the 100th item is $C(101) - C(100) = \$0.13$