Chapter 3 Solutions
MATH 251, CALCULUS I, FALL 2018

Section 3.9

4. $A = lw$ and both l and w are functions of t. Use the product rule $\frac{dA}{dt} = 140cm^2/s$.

8. (a) $0.3cm^2/min$ (b) $0.3 + \frac{3}{4}\sqrt{3}cm^2/min$ (c) $(\frac{15}{8}\sqrt{3} + \frac{3}{4}\sqrt{3} + 0.3)$
   In parts (b) and (c) you have to be careful with your derivatives. e.g in (b) b is no longer a constant so we have to use the product rule as
   $$\frac{dA}{dt} = \frac{1}{2}ab\cos\theta\frac{d\theta}{dt} + \frac{1}{2}a\sin\theta\frac{db}{dt}$$

9. (a) 1 (b) 25.
   Note: In part (b) solve for x first, then take the derivatives as usual.

12. $\frac{dx}{dt} = 6cm/s$. Note: Remember to use the product rule on the left hand side. x and y are both functions of t!

14. The diameter decreases at a rate of $\frac{1}{20\pi}cm/min$. Note: Recall from class that the surface area $S = 4\pi r^2$, if we let x be the diameter of the snowball, then $r = \frac{1}{2}x$ so we can write $S = \pi x^2$ then take the derivative, here both S and x are functions of t.

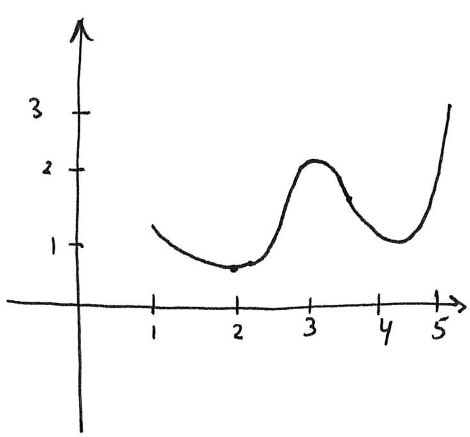
17. $65mi/hour$. This problem is similar to the boat problem we saw on web-work.

29. $\frac{6}{5\pi}$. In class we did an cone shaped tank that was emptying. Now the cone shape is growing.

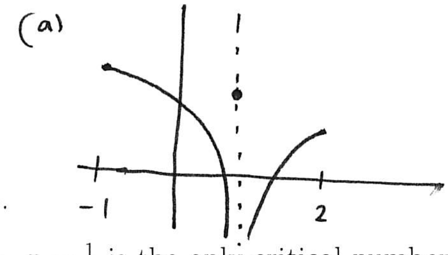
Section 4.1

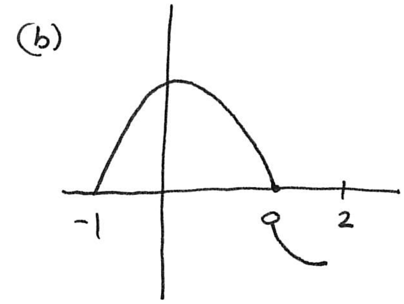
4. Absolute maximum at r; absolute minimum at a; local maxima at b and r local minimum at d neither a maximum nor a minimum at c and s.

6. There is no absolute maximum value; absolute minimum value is $g(4) = 1$; local maximum values are $g(3) = 4$ and $g(6) = 3$; local minimum values are $g(2) = 2$ and $g(4) = 1$

7. 

13.





29. $x = \frac{1}{3}$ is the only critical number.

34. $t = \frac{4}{3}$ is a critical number. Sketch the graph, notice that here you need to break up $g(t)$ into a piecewise function as

$$g(t) = \begin{cases} 3t - 4 \text{ if } t \geq \frac{4}{3} \\ 4 - 3t \text{ if } t < \frac{4}{3} \end{cases}$$

and there is a sharp corner in the graph at $\frac{4}{3}$.

44. $x = e^{\frac{1}{2}}$ is the only critical point because, even though $f'(0)$ is not defined, it is not in the domain of f.

49. $f(-1) = 8$ is the absolute maximum value and $f(2) = -19$ is the absolute minimum.

50. $f(0) = 5$ is the absolute max and $f(-3) = -76$ is the absolute minimum.

60. $f(1) = e^{1/2}$ is the absolute max and $f(-2) = -2/e$ is the absolute min.

61. $f(1) = \ln 3$ is the absolute max and $f(-\frac{1}{2}) = \ln(\frac{4}{4})$ is the absolute minimum. Remember to use the chain rule properly to find $f'(x)$.

Section 4.2

3. (a) g satisfies the hypothesis because it is continuous on $[0, 8]$ and it is differentiable on $(0, 8)$. (b) Here you will have to estimate $g(c) = \dfrac{g(8) - g(0)}{8 - 0}$ when $c \approx 2.2$ and 6.4. (c) $c \approx 3.7$ and 5.5.

5. f is a polynomial so it's continuous and differentiable. Since $f(-1) = 11$ and $f(3) = 11$, f satisfies all the hypotheses of Rolle's theorem. $f'(c) = 4c - 4$ so $f'(c) = 0$ when $c = 1$.

11. Here f is also continuous and differentiable on the interval. $f'(c) = \frac{f(b)-f(a)}{b-a} = 1$ so we can find c such that $f'(c) = 1$ You should get $c = 1$.

19. First notice that $f(-\pi) = -2\pi - 1 < 0$ and $f(0) = 1$ so f has at least one root. Now to show that it has exactly one root, we suppose that f has 2 roots say at a and b. By Rolle's Theorem there would be a number c in (a, b) such that $f'(r) = 0$ but $f'(r) = 2 - \sin r > 0$ since $\sin r \leq 1$ so r does not exist so f cannot have two distinct roots.

21. The solution proceeds as in 19, if we assume that $f(x)$ has 2 roots in the interval $[-2, 2]$ there would need to be a point r such that $f'(r) = 0$, in this interval. $f'(r) = 3r^2 - 15$ is zero at $r = \pm\sqrt{5}$ which is not in the interval in question.

25. By the mean value theorem $\frac{f(4)-f(1)}{4-1} = f'(c)$ for some $c \in (1, 4)$. But for every $c \in (1, 4)$ we have $f'(c) \geq 2$ so we can substitute $f(1) = 10$ so get

$$f(4) = f(1) + f'(c)(4 - 1) = 10 + 3f'(c) \geq 10 + 3 \cdot 2 = 16.$$

Section 4.3

1. (a) f is increasing on $(1,3)$ and $(4,6)$. (b) f is decreasing on $(0,1)$ and $(3,4)$ (c) f is concave up on $(0,2)$ (d) f is concave down on $(2,4)$ and $(4,6)$ (e) point of inflection is $(2,3)$.

2. f is increasing on $(0,1)$ and $(3,7)$ (b) f is decreasing on $(1,3)$ (c) f is concave up on $(2,4)$ and $(5,7)$. (d) f is concave down on $(0,2)$ and $(4,5)$. (e) inflection points $(2,2)$, $(4,3)$ and $(5,4)$.

6. Notice that this is the graph of $f'(x)$! so (a) $f'(x) > 0$ and f is increasing on $(0,1)$ and $(5,7)$. f is decreasing on $(1,5)$ and $(7,8)$. (b) Since $f'(x) = 0$ at $x = 1$ and $x = 7$ and f' changes from positive to negative on both values we have a local maxima at $x = 1$ and $x = 7$ and local min at $x = 5$. Here we are using the first derivative test.

8. (a) f is increasing on $(0,4)$ and $(6,8)$. (b) f has a local maximum where it changes from increasing to decreasing so $x = 4$ and $x = 8$. local min at $x = 6$. (c) f is concave up where f' is increasing (hence $f''(x) > 0$). This happens on $(0,1),(2,3)$ and $(5,7)$. Similar reasoning for concave down on $(1,2)$, $(3,5)$ and $(7,9)$. (d) The inflection point happens where concavity changes – This happens at $x = 1, 2, 3, 5$ and 7.

11. $f'(x) = 4x(x+1)(x-1)$ so f is decreasing on $(-\infty,-1)$ and $(0,1)$ and increasing on $(-1,0)$ and $(1,\infty)$. (b) local max at $x = 0$ and local min at $x = \pm 1$. (c) Inflection points at $(\pm\sqrt{3}/3, \frac{22}{9})$.

12. (a) $f'(x) = \frac{1-x^2}{(x^2+1)^2} = -\frac{(x+1)(x-1}{(x^2+1)^2}$ Using the table with critical points at $x = -1, 1$ we can get f increases on $(-1,1)$ and f is decreasing on $(-\infty,-1)$ and $(1,\infty)$. (b) $x = -1$ is a local min and $x = 1$ is a local max. (c) $f''(x) = \frac{2x(x^2-3)}{(x^2+1)^3}$ so we have inflection points at $x = 0, \pm\sqrt{3}$.

15. $f'(x) = 2e^{2x} - e^{-x}$ so f is increasing on $(-\frac{1}{3}\ln 2, \infty)$ and decreason on $(-\infty, -\frac{1}{3}\ln 2)$. (b) $x = -\frac{1}{3}\ln 2$ is a local min point. (c) There are no points of inflection

17. (a) f is increasing on $(1,\infty)$ and decreasing on $(0,1)$ (b) $f(1) = 0$ is a local min value (c) There is no inflection point!