Exam 1 will cover Numerical Differentiation and Integration.


You are responsible for all material covered in class and problems assigned for homework.

I expect you to be able to:

1. **Numerical Differentiation**
   (a) Derive polynomial approximations of $f$ via *Taylor expansion*.
   (b) Write down the *Lagrange interpolating polynomial* of a function $f$ given data points.
   (c) Derive numerical differentiation formulas and error terms using Taylor series.
   (d) Understand (and define) the concept of local truncation error.
   (e) Understand the impact of roundoff error on numerical differentiation formulas.
   (f) Understand the concept of order of convergence of a numerical differentiation formula.

2. **Numerical Integration**
   (a) Understand how *Newton-Cotes* methods are derived.
   (b) Give two examples of *Newton-Cotes* methods and their order of convergence.
   (c) Use the *method of undetermined coefficients* to derive quadrature rules.
   (d) Define the degree of precision.
   (e) Find the degree of precision of a quadrature rule.
   (f) Understand the concept of *Composite Newton-Cotes methods*.
   (g) What is the advantage of *Composite Newton-Cotes methods*?
   (h) Determine the number of subintervals necessary to attain a specified level of approximation of an integral.
   (i) Understand how *Gaussian quadrature methods* are derived.
   (j) In general, what makes *Gaussian methods* superior to *Newton-Cotes methods*?
   (k) Explain why *Composite* methods work better than expected for periodic integrals.

**Some practice problems:**

1. Derive the second order central difference approximation of the derivative including the error formula

   \[ f'(x) = \frac{f(x + h) - f(x - h)}{2h} - \frac{h^2}{6} f'''(c), \quad c \in (x - h, x + h) \]

2. Derive

   \[ f''(x) = \frac{f(x - 2h) - f(x - h) + f(x)}{h^2} \]

   What is the error term?
