Exam 2 will cover initial value problems

**Reading:** Chapter 11: 11.1, 11.2, 11.3.1-11.3.3, 11.4.1-11.4.2
You are responsible for all material covered in class and problems assigned for homework.

**Some problems to think about:**

1. State the conditions that need to be satisfied by the initial value problem

   \[ y'(t) = f(t, y(t)) \]
   \[ y(t_0) = y_0 \]
   \[ a \leq t \leq b \]

   in order for the problem to have

   (a) A unique local solution
   (b) A unique global solution
   (c) More than one solution

2. What does it mean for an initial value problem to be *wellposed*?

3. Derive Euler’s method from Taylor’s theorem

4. Define the following terms:
   (a) local truncation error
   (b) consistent method
   (c) convergent method
   (d) global error

5. What is the relationship between the global error and the local truncation error?

6. State the local truncation error for Euler, RK2, RK4

7. What makes high order methods (e.g. RK2, RK4) more efficient compared to Euler’s method?

8. State 2 advantages of multi-step methods

9. What is the difference between high order Taylor methods and RK methods?

10. What is a *stiff* initial value problem

11. What is the region of absolute stability of an initial value problem?

12. Show that the region of absolute stability for Euler’s method is the unit circle centred at \((-1, 0)\)

13. What is a *A*-stable method?

14. Show that the Backward Euler method and Trapezoidal methods are A-stable.