Intermediate Value Theorem

**Theorem**

*Intermediate Value Theorem* Suppose \( f \) is continuous on \([a, b]\) and let \( N \) be any number between \( f(a) \) and \( f(b) \), where \( f(a) \neq f(b) \). Then there exists a number \( c \) in \((a, b)\) such that \( f(c) = N \).
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**Generalized Intermediate Value Theorem**

Theorem

Let $f$ be continuous on $[a, b]$. Let $x_0, x_1, \ldots, x_n$ be points in $[a, b]$ and $a_1, a_2, \ldots, a_n > 0$. There exists a number $c$ between $a$ and $b$ such that

$$(a_1 + \cdots + a_n)f(c) = a_1f(x_1) + \cdots + a_nf(x_n)$$
Generalized IVT applied to error estimates

Recall

\[ f'(x) = \frac{f(x + h) - f(x - h)}{2h} - \frac{h^2}{12} \left( f'''(c_1) + f'''(c_2) \right) \]

for \( c_1 \in (x, x + h) \) and \( c_2 \in (x - h, x) \).
Generalized IVT applied to error estimates

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Let \( f \) be continuous on \([a, b]\). Let \( x_0, x_1, \ldots, x_n \) be points in \([a, b]\) and \( a_1, a_2, \ldots, a_n > 0 \). There exists a number \( c \) between \( a \) and \( b \) such that

\[
(a_1 + \cdots + a_n)f(c) = a_1 f(x_1) + \cdots + a_n f(x_n)
\]

We can combine the error terms of the central difference formula as

\[
\left( \frac{1}{12} + \frac{1}{12} \right)f(c) = \frac{h^2}{12} \left( f'''(c_1) + f'''(c_2) \right)
\]

for \( c \in (x - h, x + h) \) to obtain a nicer looking estimate:

\[
f'(x) = \frac{f(x + h) - f(x - h)}{2h} - \frac{h^2}{6} f'''(c) \quad c \in (x - h, x + h)
\]