Some solutions to Homework 8 & 9
MATH 251, CALCULUS I, FALL 2013

Section 4.4,14.

$$\lim_{x\to 0}\frac{x^2}{1-\cos x}$$

First notice that this limit is of the form $\frac{0}{0}$ so we apply *l'hopital's Rule* twice.

$$\lim_{x\to 0}\frac{x^2}{1-\cos x}=\lim_{x\to 0}\frac{2x}{\sin x}=\lim_{x\to 0}\frac{2}{\cos x}=2$$

Section 4.3,61.

$$\lim_{x\to\infty} x^{1/x}$$

This is also an indeterminate limit of the form ∞^0 so again we use *l'hopital's Rule!*.

$$y=x^{1/x}\Longrightarrow \ln y=(1/x)\ln x\Longrightarrow \lim_{x\to\infty}\ln y=\lim_{x\to\infty}\frac{\ln x}{x}=\lim_{x\to\infty}\frac{1/x}{1}=0$$
$$\Longrightarrow \lim_{x\to\infty}x^{1/x}=\lim_{x\to\infty}e^{\ln y}=e^0=1$$

Section 4.7,20. Find the point on the curve $y=\sqrt{x}$ that is closest to the point $(3,0)$.
The distance d from $(3,0)$ to a point $(x,\sqrt{x})$ on the curve is given by

$$d=\sqrt{(x-3)^2+(\sqrt{x}-0)^2}$$

and as we discussed in class the critical points of the square of the distance are the same as those of the distance but it is easier to work with the square of the distance

$$S(x)=d^2=(x-3)^2+x$$

To obtain the critical points we find

$$S'(x)=2(x-3)+1\Longrightarrow x=-\frac{5}{2}.$$

Now we have one critical point we have to check that this is a maximum:

$$S''(x)=2>0\Longrightarrow$$

therefore the critical point $x=\frac{5}{2}$ is a minimum. Thus, the y value is $\sqrt{\frac{5}{2}}$ and the point is $(\frac{5}{2},\sqrt{\frac{5}{2}})$.

Section 5.3,34.

$$\int_0^3(2\sin x-e^x\,dx=-2\cos x-e^x\big|_0^3=(-2\cos 3-e^3)-)-(-2-1)=3-2\cos 3-e^3$$

Section 5.3,40.

$$\int_1^2 \frac{4+u^2}{u^3}\,du = \int_1^2 4u^{-3} + u^{-1}\,du = \frac{4}{-2}u^{-2} + \ln|u|\Big|_1^2 = \frac{3}{2} + \ln 2$$

Section 5.3,57.

$$F(x) = \int_x^{x^2} e^{t^2}\,dt = \int_x^0 e^{t^2}\,dt + \int_0^{x^2} e^{(t^2)}\,dt = -\int_0^x e^{t^2}\,dt + \int_0^{x^2} e^{t^2}\,dt \Longrightarrow$$

$$F'(x) = -e^{x^2} + e^{(x^2)^2}\cdot\frac{d}{dx}(x^2) = -e^{x^2} + 2xe^{x^4}$$

Section 5.4,11.

$$\int \frac{x^3 - 2\sqrt{x}}{x}\,dx = \int (\frac{x^3}{x} - \frac{2x^{1/2}}{x})\,dx = \int (x^2 - 2x^{-1/2}\,dx = \frac{x^3}{3} + 2\frac{x^{1/2}}{1/2} + C$$

Section 5.4,19.

$$\int \cos x + \frac{1}{2}x = \sin x + \frac{1}{4}x^2 + C$$

Section 5.5,8.

$$\int x^2 e^{x^3}\,dx$$

Let $u = x^3$ then $du = 3x^2dx$ and $x^2dx = \dfrac{1}{3}$ so

$$\int x^2 e^{x^3}\,dx = \int e^u(\frac{1}{3}du) = \frac{1}{3}e^u + C = \frac{1}{3}e^{x^3} + C$$

Section 5.5,27.

$$\int (x^2+1)(x^3+3x)^4\,dx$$

Let $u = x^3 + 3x$ then $du = 3x^2 + 3)dx$ and $\dfrac{1}{3}du = (x^2+1)dx$ so

$$\int (x^2+1)(x^3+3x)^4\,dx = \int u^4\frac{1}{3}du = \frac{1}{3}\cdot\frac{1}{5}u^5 + C = \frac{1}{15}(x^3+3x)^5 + C$$

Section 5.5,32

$$\int \frac{\sin(\ln x)}{x}\,dx$$

Let $u = \ln x$. Then $du = (1/x)dx$, so

$$\int \frac{\sin(\ln x)}{x}\,dx = \int \sin u\,du = -\cos u + C = -\cos(\ln x) + C$$