Quiz 2
Name: ***Solutions***
Math 251.03(04), Calculus I, Fall 2013

1. Find the equation of the tangent line to $y = \dfrac{1}{1-x}$ at $P = (2, -1)$

   Hint: The slope of the tangent line is the limit of the slopes of carefully chosen secant lines passing through P

   The slope of the tangent line is the limit of the slopes of secant lines passing through $P = (2, -1)$ and $Q = \left(x, \dfrac{1}{1-x}\right)$.

   First we write a function for the slope of each secant line $\overline{PQ}$

$$m(x) = \frac{\dfrac{1}{1-x} + 1}{x-2} = \frac{\dfrac{-(x-2)}{1-x}}{x-2} = \frac{-1}{1-x}$$

   Then the slope of the tangent line is

$$\lim_{x\to 2} \frac{-1}{1-x} = \frac{-1}{1-2} = 1$$

   The point slope form of the equation of the tangent line is:

$$y + 1 = 1(x-2)$$

2. Find the limit if it exists. If not say why.

$$\lim_{x\to 3}(2x + |x-3|)$$

   Let $g(x) = 2x + |x-3|$. Using the definition of the absolute value function we note that

$$g(x) = \begin{cases} 2x + (x-3) & \text{if } x \geq 3 \\ 2x - (x-3) & \text{if } x < 3 \end{cases}$$

   Therefore

$$\lim_{x\to 3^+} g(x) = \lim_{x\to 3^+}(2x + (x-3)) = 2(3) + (3-3) = 6$$

   and

$$\lim_{x\to 3^-} g(x) = \lim_{x\to 3^-}(2x - (x-3)) = 2(3) - (3-3) = 6$$

   We conclude that

$$\lim_{x\to 3} g(x) = 6$$

   since the left and right hand limits agree!

3. Calculate

$$\lim_{t\to 0}\left(\frac{1}{t\sqrt{1+t}}-\frac{1}{t}\right)$$

$$\begin{aligned}
\lim_{t\to 0}\left(\frac{1}{t\sqrt{1+t}}-\frac{1}{t}\right) &= \lim_{t\to 0}\left(\frac{1-\sqrt{1+t}}{t\sqrt{1+t}}\right)\\
&= \lim_{t\to 0}\left(\frac{1-\sqrt{1+t}}{t\sqrt{1+t}}\cdot\frac{1+\sqrt{1+t}}{1+\sqrt{1+t}}\right)\\
&= \lim_{t\to 0}\frac{1^2-(\sqrt{1+t})^2}{(t\sqrt{1+t})(1+\sqrt{1+t})}\\
&= \lim_{t\to 0}\frac{-t}{(t\sqrt{1+t})(1+\sqrt{1+t})}\\
&= \lim_{t\to 0}\frac{-1}{(\sqrt{1+t})(1+\sqrt{1+t})}\\
&= -\frac{1}{2}
\end{aligned}$$

4. Skecth the graph of a function with the following properties

   (a) The limit of the function exists at $x = 2$ but $x = 2$ is not in the domain of the function.

   (b) The function is defined at $x = 0$ but the limit does not exist at $x = 0$.