**Quiz 3**
**Name: Quiz 3 solutions**
**Math 251, Calculus I, Fall 2013**

1. Use the Intermediate value theorem to show that $\sqrt[3]{x} = 1 - \sqrt{x}$ has a root on $(0, 1)$. *First notice that the function* $f(x) = \sqrt[3]{x} - 1 + \sqrt{x}$ *is continuous on the interval* $[0, 1]$ *therefore we know that by the intermediate value theorem the function has to assume all intermediate values between* $f(0)$ *and* $f(1)$*. We check*

$$f(0) = 0 - 1 + 0 = -1$$
$$f(1) = 1 - 1 + 1 = 1$$

*Since* $-1 < 0 < 1$ *there exists a point* $c$ *in* $[0, 1]$ *such that* $f(x) = 0$ *by the intermediate value theorem therefore the function* $y = f(x)$ *has a root on the interval* $(0, 1)$.

2. Find $\lim_{x\to\infty} e^{-2x}\cos(x)$ *Since* $-1 \leq \cos(x) \leq 1$ *and* $e^{-2x} > 0$ *we have*

$$-e^{-2x} \leq e^{-2x}\cos(x) \leq e^{-2x}$$

*We know that* $\lim_{x\to\infty} e^{-2x} = 0$ *and* $\lim_{x\to\infty} -e^{-2x}$ *therefore by the Squeeze theorem* $\lim_{x\to\infty} e^{-2x}\cos(x) = 0$

3. Use the defintion of the derivative to find $f'(x)$ for $f(x) = \dfrac{1}{x}$. *Recall the defintion of the derivative*

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

*Therefore:*

$$\begin{aligned} f'(x) &= \lim_{h\to 0}\left(\frac{\frac{1}{x+h} - \frac{1}{x}}{h}\right) \\ &= \lim_{h\to 0}\left(\frac{\frac{x-(x+h)}{x(x+h)}}{h}\right) \\ &= \lim_{h\to 0}\left(\frac{\frac{-h}{x(x+h)}}{h}\right) \\ &= \lim_{h\to 0}\left(\frac{-1}{x(x+h)}\right) = -\frac{1}{x^2} \end{aligned}$$