

Name:

Functions and Change Solutions

Math 351.01, Calculus III, Spring 2016

Section 1.1 - 11,13,15,25,30,33

11 Substituting  $x = 5$  into  $f(x) = 10x - x^2$  gives

$$f(5) = 10(5) - (5)^2 = 50 - 25 = 25.$$

13 Looking at the graph, we see that the point on the graph with an x-coordinate of 5 has a y-coordinate of 2. Thus

$$f(5) = 2$$

15 (a) We are asked for the value of  $y$  when  $x$  is zero. That is, we are asked for  $f(0)$ . Plugging in we get

$$f(0) = (0)^2 + 2 = 0 + 2 = 2.$$

(b) Substituting we get

$$f(3) = (3)^2 + 2 = 9 + 2 = 11$$

(c) Asking what values of  $x$  give a  $y$ -value of 11 is the same as solving

$$y = 11 = x^2 + 2x^2 = 9 \longrightarrow x = \pm\sqrt{9}$$

(d) No. No matter what,  $x^2$  is greater than or equal to 0, so  $y = x^2 + 2$  is greater than or equal to 2.

25 (a) The original deposit is the balance,  $B$ , when  $t = 0$ , which is the vertical intercept. The original deposit was \$1000.

(b) It appears that  $f(10) \approx 2200$ . The balance in the account after 10 years is about \$2200.

(c) When  $B = 5000$ , it appears that  $t \approx 20$ . It takes about 20 years for the balance in the account to reach \$5000.

30 Your plot should be a line with positive slope, increasing return comes with increasing risk.

33 (a) i. I The incidence of cancer increase with age, but the rate of increase slows down slightly.

The graph is nearly linear. This type of cancer is closely related to the aging process

ii. II In this case a peak is reached at about age 55, after which the incidence decreases.

iii. III This type of cancer has an increased incidence until the age of about 48, then a slight decrease, followed by a gradual increase

iv. IV In this case the incidence rises steeply until the age of 30, after which it levels out completely.

v. V This type of cancer is relatively frequent in young children, and its incidence increases gradually from about the age of 20.

vi. VI This type of cancer is not age-related all age-groups are equally vulnerable, although the overall incidence is low (assuming each graph has the same vertical scale).

(b) Graph (V) shows a relatively high incidence rate for children. Leukemia behaves in this way.

(c) Graph (III) could represent cancer in women with menopause as a significant factor. Breast cancer is a possibility here

- (d) Graph (I) shows a cancer which might be caused by toxins building up in the body. Lung cancer is a good example of this.

**Section 1.2 – 7,12,14,16,17,33**

- 7 Rewrite the equation of the line as  $y = 2x - \frac{2}{3}$ . The slope is 2 and intercept is  $-\frac{2}{3}$ .
- 12
- The first company  $C_1(m) = 40 + 0.15m$  and the competitor's price is  $C_2(m) = 50 + 0.1m$ .
  - Use the slope intercept form to plot.
  - To find which one is cheaper, we need to determine where the two lines intersect. Set  $C_1 = C_2$  and solve. You should obtain  $m = 200$ . This means that if you want to travel less than 200, the first car is cheaper.
- 14 (a) and (b) have constant rates of change, therefore they are linear. (c) is not.
- 16 This is a linear function with vertical intercept 25 and slope 0.05. The formula for the monthly charge is  $C = 25 + 0.05m$ .
- 17 (a) The slope is 1.8 billion dollars per year. McDonalds revenue is increasing at a rate of 1.8 billion dollars per year.
- (b) The vertical intercept is 19.1 billion dollars. In 2005, McDonalds revenue was 19.1 billion dollars.
- (c) Substituting  $t = 10$ , we have  $R = 19.1 + 1.8 * 10 = 37.1$  billion dollars.
- (d) Substitute  $R = 35$  and solve for  $t$ . You should obtain  $t = 8.83$  years.
- 33 (a) When the unemployment rate is constant, Okuna law states that US production increases by 3.5% annually.
- (b) When unemployment rises from 5% to 8% we have  $u = 3$  and therefore  $y = 3.5 - 2u = -2.5$  so the national production will decrease by 2.5%.
- (c) If annual production does not change, then  $y = 0$ . Hence  $0 = 3.52u$  and so  $u = 1.75$ . There is no change in annual production if the unemployment rate goes up 1.75%.
- (d) The coefficient 2 is the slope  $\frac{\Delta y}{\Delta u}$ . Every 1% increase in the unemployment rate during the course of a year results in an additional 2% decrease in annual production for the year.

**Section 1.3 – 7,8,10,11,33,55**

- 7 The function is increasing and concave up between D and E, and between H and I. It is increasing and concave down between A and B, and between E and F. It is decreasing and concave up between C and D, and between G and H. Finally, it is decreasing and concave down between B and C, and between F and G.
- 8 Average rate of change is  $\frac{f(3) - f(1)}{3 - 1} = \frac{18 - 2}{2} = 8$
- 10 When  $t = 0$ , we have  $B = 1000(1.08)^0 = 1000$ . When  $t = 5$ , we have  $B = 1000(1.08)^5 = 1469.33$ . We have an average rate of change of  $= \frac{\Delta B}{\Delta t} = \frac{1469.33 - 1000}{5 - 0} = 9.87 \text{dollars/year}$ .
- 11 (a) Between 2008 and 2010, change in net sales =  $-134$  million dollars.

- (b) Average rate of change =  $-67$  million dollars per year  
(c) The average rate of change is positive from 2009 to 2010.

33 (a)  $f(1985) = 13$ ,  $f(1990) = 99$ .

(b) The average yearly increase is the rate of change  $\frac{f(1990) - f(1985)}{1990 - 1985} = \frac{99 - 13}{5} = 17.2$  billion-aires per year

(c) Since the rate of change is constant, we can use a linear function of the form

$$f(t) = b + 17.2t$$

where  $f(1985) = 13$  so that

$$13 = b + 17.2(1985)$$

solving for  $b$  gives  $f(t) = 17.2t - 34,129$ .

55 We did this problem as an example in class.