Homework #4

4(a) We are given $s(t) = 4t^2 + 3$

(i) Average velocity between $t=1$ and $t=1+h$ is

$$\frac{s(1+h) - s(1)}{(1+h)-1} = \frac{s(1+h)-s(1)}{h}$$

(recall that the idea of average velocity is $\frac{\text{distance covered}}{\text{time}}$)

(ii) for $h = 0.1$, Average velocity $= \frac{s(1.1)-s(1)}{1.1-1} = \frac{7.84-7}{0.1} = 8.4\,m/s$

(ii) for $h=0.01$,

Average velocity is $\frac{s(1.01)-s(1)}{1.01-1} = \frac{7.0804-7}{0.01} = 8.04\,m/s$

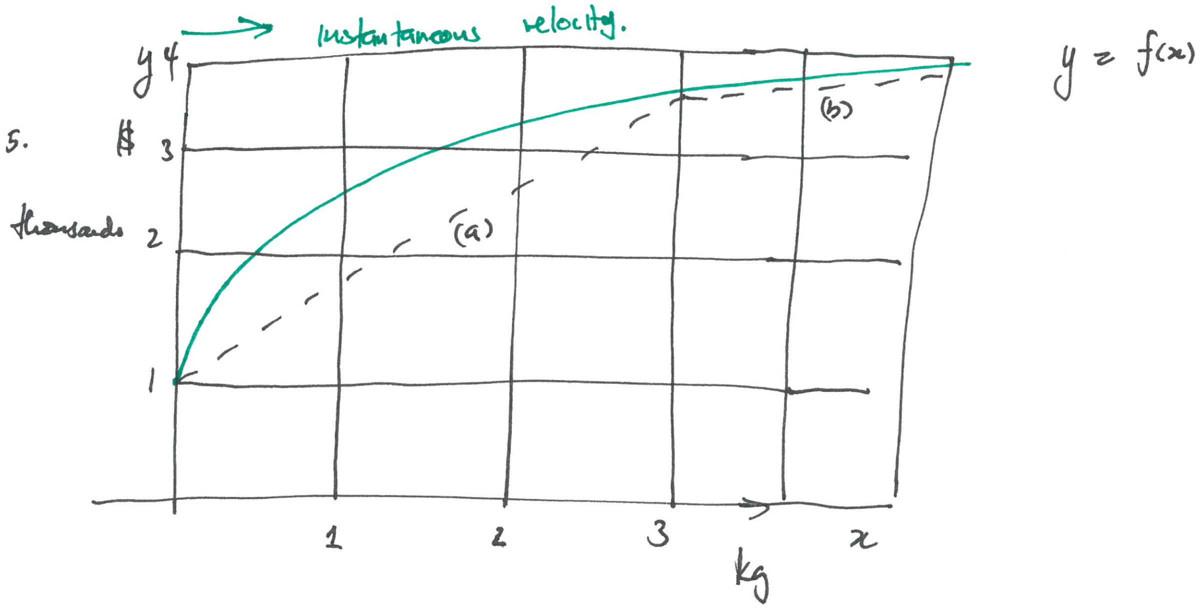
and so on. Notice that the average velocity gets closer to $8m/s$, this is the instantaneous velocity @ $t=1$

The instantaneous velocity should be $8m/s$

(b) We can now check $s'(t) = 8t$, since $s(t) = 4t^2+3$

$s'(1) = 8m/s$. $\Rightarrow$ Average velocity for smaller time intervals $\rightarrow$ instantaneous velocity.

5.



Average Rate of change between $x=0$ and $x=3$ is the slope of line (a) above

between $x=3$ and $x=5$, that is line (b). Since slope (a) > slope (b) the average rate of change of f is greater on $[0,3]$

(b) Instantaneous rate of change is greater at $x=1$ than $x=4$.

Draw the tangent line.

(c) Units $= \$/kg$.

6 (a) 7ft/s

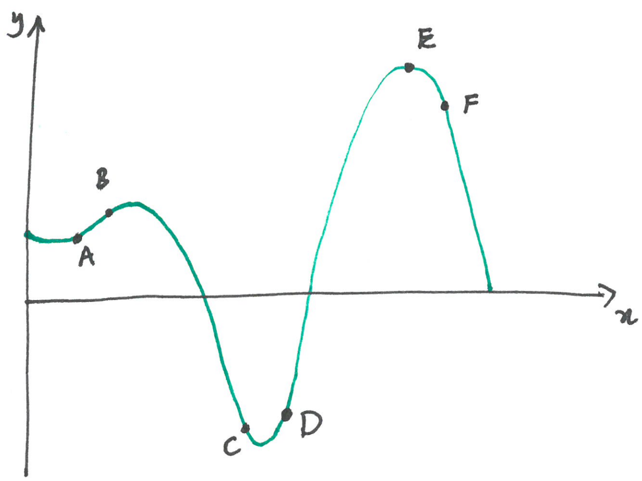
(b) use an interval of size 0.1 to estimate

Instantaneous velocity $\simeq \dfrac{s(2.1)-s(2)}{2.1-2} = \dfrac{4.41-4}{0.1} = 4.1\,m/s$

Try size $=0.01$. You should get 4.01

This means that the instantaneous velocity $\cong 4.$

12.



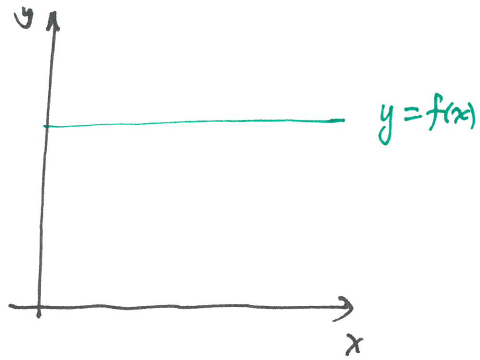
Slope	Point
−3	F
−1	C
0	E
0.5	A
1	B
2	D

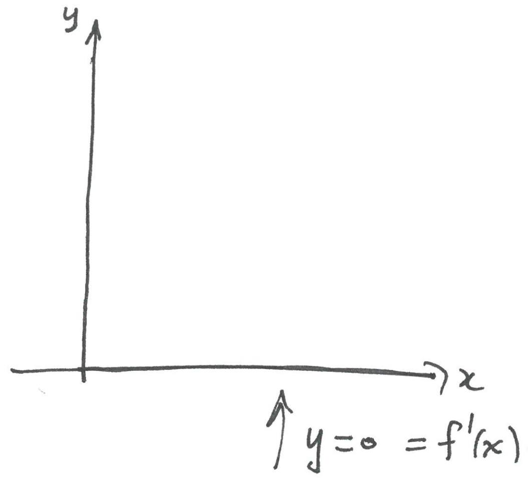
18. (a) $f(7)=3$

(b) $f'(7) =$ slope of tangent line $= \dfrac{3.8-3}{7.2-7} = \dfrac{0.8}{0.2} = 4.$

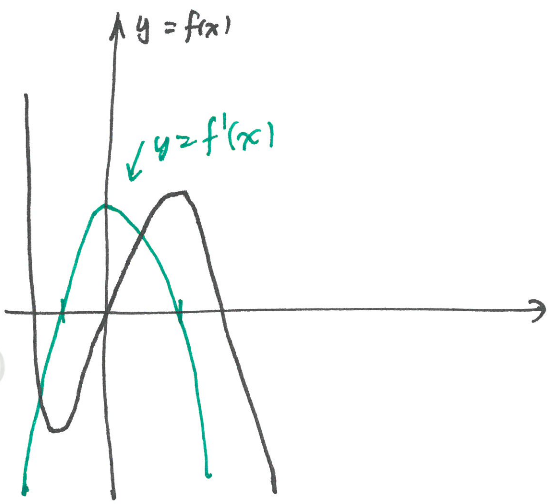
Section 2.2

12.





18.



Matches with $\overline{\underline{\text{VIII}}}$

Section 2.3

(14) $B = g(r)$ - value of investment as a function of interest rate

(a) $g(5) \simeq 1649$ means investing \$1000 yields \$1649 after 10 years

(b) $g'(5) \cong 165$

Note that $g'(5)$ is $\dfrac{dB}{dr}$ at $t=5$.

$g'(5) = 165 \Rightarrow$ the balance will increase by 165 if the interest rate increases by 1%.

i.e $g(6) \cong g(5) + 165$. Units of $g'(r)$ = dollars / percent.

16. $W = f(c)$ - is the average weight, W, in pounds as a function of the number of calories c/day

$f(1800) = 155$ means consuming 1800 cal/day results in a ~~gain~~ weight of 155.

$f'(2000) = 0$ means consuming 2000 calories per day causes no weight gain or loss.

22 $f(26) \cong f(25) + f'(25)(26-25)$
$= 3.6 + (-0.2)\cdot 1 = 3.4$

28. (a) $f(8) = 5.1 \Rightarrow$ \$5.1B in sales in 2008

$f'(8)$ - in 2008, annual net sales increase by 0.22B in the next year

(b) $f(12) \cong 5.1 + f'(8)\ (12-8)$
$= 5.1 + 0.44 \cdot 4 \approx 5.98B$

$f(12) = 5.98B \Rightarrow$ in 2012, the projected sales are 5.98B

42. We hope that $f' > 0$

(b) $f'(100) = 2$ means Each extra dollar spent on advertising results in \$2 in sales

$f'(100) = 0.5$ means each extra dollar spent on advertising results in \$0.5 in sales

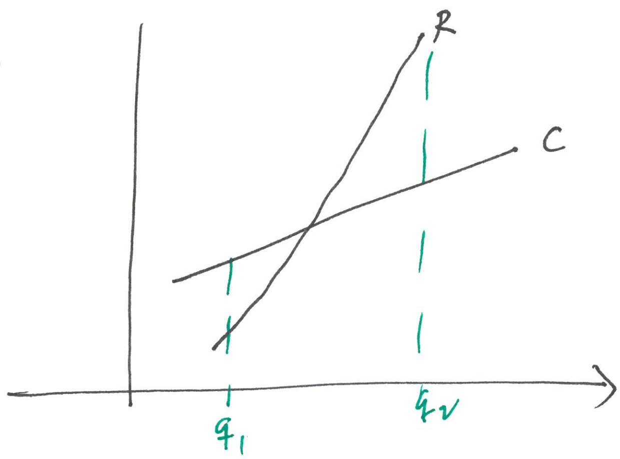
(c) $f'(100) = 2 \Rightarrow$ spend more on advertising since we gain a dollar
$f'(100) = 0.5 \Rightarrow$ do not spend more since we gain less than \$1.00 spent

SECTION 2.5

1. Approximate Marginal cost = Average increase in costs

$$MC \cong \frac{\Delta C}{\Delta q} = \frac{4830 - 4800}{1305 - 1295} = \$3/\text{item}$$

8.



@ q_1 $MC > MR$

@ q_2 $MR > MC$

12. At $q = 50$, the slope of the revenue function > slope of cost therefore a 50th bus should be added

@ $q = 90$ slope of revenue function is less than the slope of the cost function so the 90th bus should not be added.