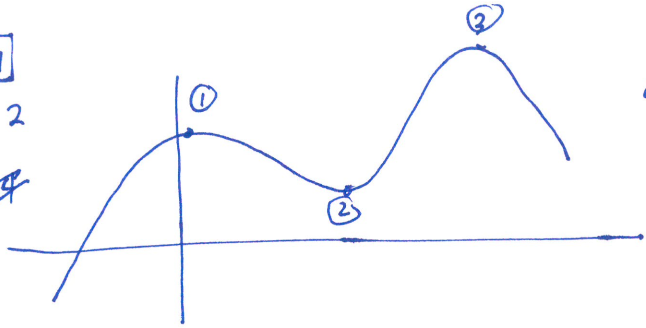
SOLUTIONS TO EVEN PROBLEMS

$\boxed{4.1}$

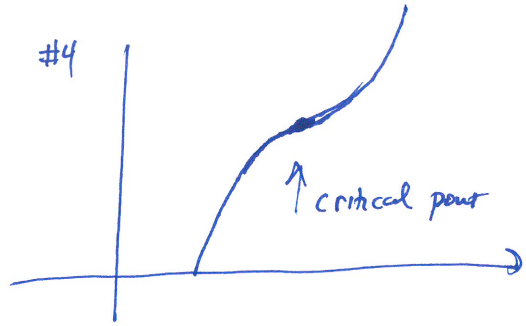
\# 2

\# ~~4~~



1 - 3 are critical pts.

\#4



\#20

$f(x) = x^4 - 4x^3 + 8x$

$f'(x) = 4x^3 - 12x^2 + 8$, we can check $f'(1) = 4 - 12 + 8 = -8 + 8 = 0$

so $x = 1$ is a critical point.

To see what type of point, evaluate $f''(x) = 12x^2 - 24x$

Since $f''(1) = 12 - 24 < 0$ this point is a local max.

\#24 (a) Increasing for $x > 0$, decreasing for $x < 0$.

(b) $f(0)$ is a local and global min

f has no global max. [because $f' > 0$ for all $x > 0$ so the function continues to increase

\#26. (a) decreasing for $x < 0$, increasing for $0 < x < 4$, decreasing for $x > 4$

$f(0)$ is a local ~~max~~ min

$f(4)$ is a local max

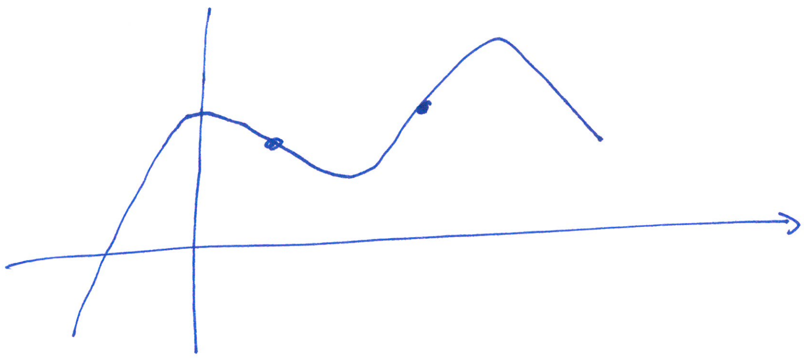
} Use the 1st derivative test to check this.

\#~~38~~. I removed this problem.

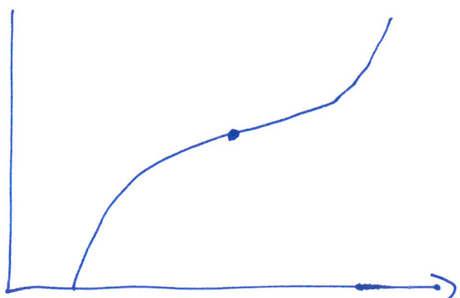
4.2

#2.

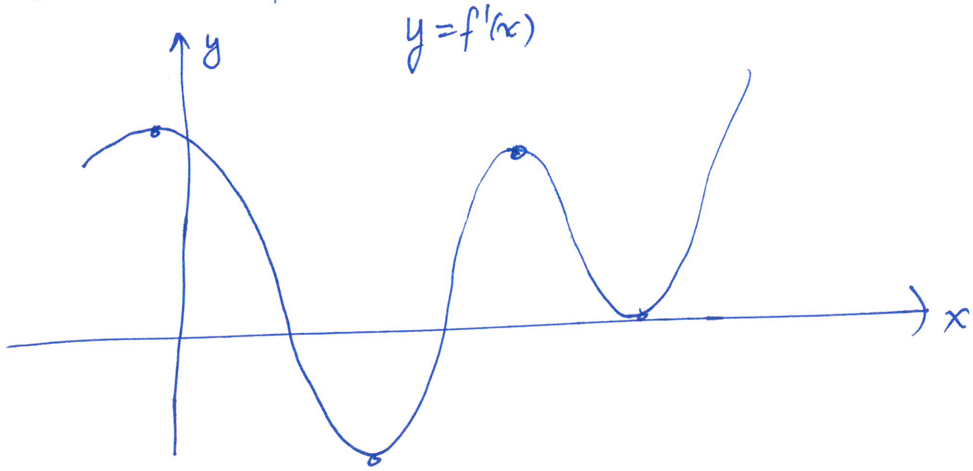
We have inflection points where the concavity changes



#4.



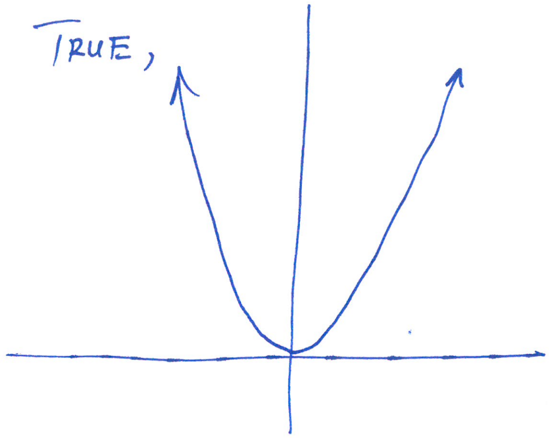
#24 To find inflection points we need to find places were f'' changes sign. Since we have the graph of $y = f'(x)$, f'' changes sign at max / min points of $f'(x)$



Inflection pts.

4.3

#8 TRUE,



the max is always at the endpoints.

#18 $f(x) = x^3 - 3x^2 - 9x + 15.$

(a) $f'(x) = 3x^2 - 6x - 9$

$f''(x) = 6x - 6$

(b) Critical points

$f'(x) = 0 \Rightarrow 3x^2 - 6x - 9 = 0$

$3(x^2 - 2x - 3) = 0$

$3(x+1)(x-3) = 0$

$x = -1, x = 3$ are critical points

(c) Inflection point

$f''(x) = 0 \Rightarrow 6x - 6 = 0$

$x = 1$

We need to check that $f''(x)$ changes sign at $x = 1$

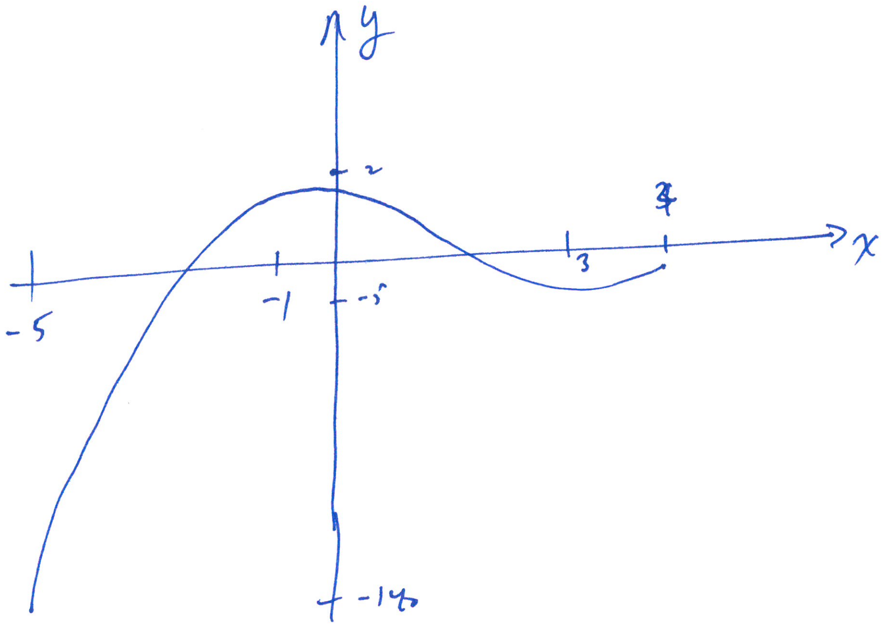
$f''(x) < 0$ | $f'' > 0$

$x = 1$

$f''(x) = 6(x-1)$

(d) $f(-5) = -140$
$f(4) = -5$
$f(-1) = 2$
$f(3) = -12$

so global max $= 2$ at $x = -1$
global min $= -140$ at $x = -5$



4.4

2. Profit function

$$\pi(q) = R(q) - C(q)$$
$$= (500q - q^2) - (150 + 10q)$$
$$= 490q - q^2 - 150.$$

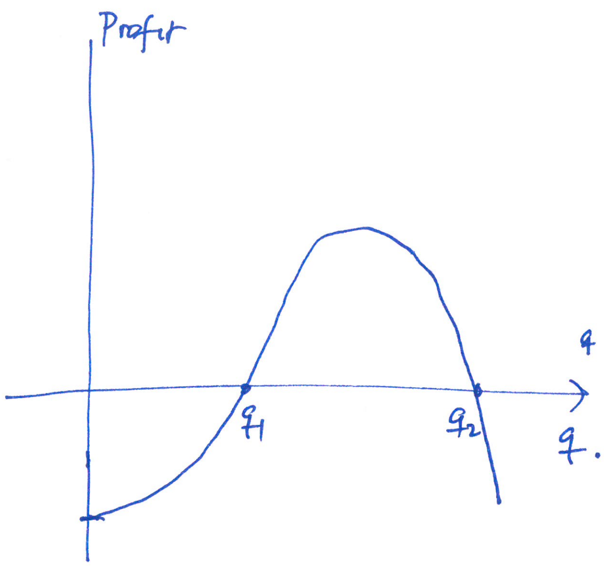
Maximum profit occurs when $\pi'(q) = 0$

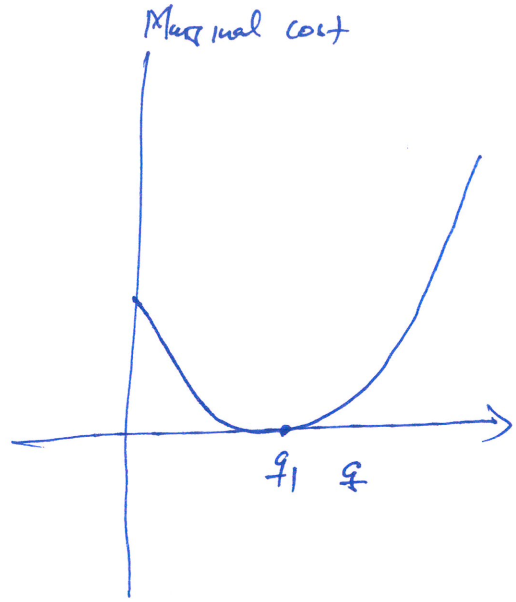
$\pi'(q) = 0 \Rightarrow 490 - 2q = 0$ so $q = 245$ items.

Check $\pi''(q) = -2$, so this is indeed a max.

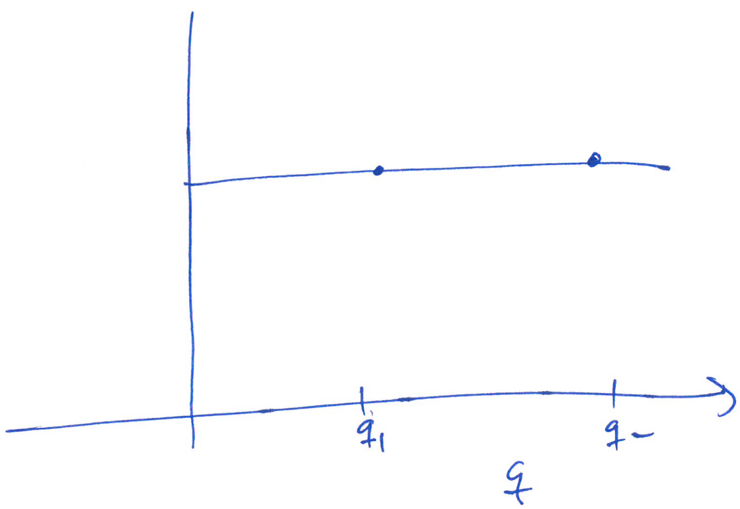
4.4

#4





Marginal Revenue



6 (a) $C(0)$ – ~~te~~ This represents fixed costs.

(b) Marginal cost decreases slowly, and then increases as q increases

(c) Concave down means Marginal cost is decreasing

(d) Inflection point is a point of minimum/maximum marginal cost

(e) No, cost functions depend on the product.

4.4

#18. (a) Revenue = Price × quantity

$(45-0.01q)\cdot q = 45q - 0.01q^2.$

(b) Max Revenue occurs when $R'(q) = 0$

In this case $R'(q) = 45 - 0.02q = 0$

$q = \underline{2250}$

(c) $p = \$22.50$

(d) Total Revenue $= \underline{\$50{,}625}$

20 (a) A \$4.00 demand = 4000 units.

For every \$0.25 increase in price, the demand increases by 200 units

$$q = 4000 - \frac{(4-p)}{0.25}(200)$$

start @ 4000

this ensures that every 0.25 results in a decrease in demand of $\underline{200}$. check!

Simplifying

$$q = 4000 + 800(4-p)$$
$$= 7200 - 800p.$$

(b) Revenue $= qp = (7200 - 800p)\,p = 7200p - 800p^2$

$R'(p) = 0 \Rightarrow$ max revenue.

$p = 4.5$ and Max Revenue is $\underline{\$16,200}$

#22

(a) $\pi = R - C$

$(-5q + 4000)q - (6q + 5)$

(b) Max Profit

$\pi'(q) = 0$ and solve

$q = 399.4$

(c) For $q = 399.4$

the max profit is $\pi(399.4)$ plug into $\pi(q)$

$= \underline{797,596.80}$

SECTION 4.5

2. (a) $a(q) = \$1.60/\text{unit}$ (b) $a(q) = \dfrac{C(q)}{q}$.

(c) See plots from class. $a(q)$ is minimum when $a(q) = C'(q)$.

4. (a) \$12

(b) \$14.50.

8. (a) \$24,000 is monthy revenue so profit is 24,000 − 2400 so yes you are making money

(b) Yes MR > MC

(c) Increase production.

10. (a) $MC = \$10$, which is below average cost so increasing production decreases average cost

(b) It is not possible to tell since $\pi = R - C$, we have no revenue information