1. The Figure above shows the cost, \( y = f(x) \), of manufacturing \( x \) kilograms of a chemical.

(a) Is the average rate of change of the cost greater between \( x = 0 \) and \( x = 3 \), or between \( x = 3 \) and \( x = 5 \)? Explain your answer graphically.

The average rate of change of a function over an interval is represented graphically as the slope of the secant line to its graph over the interval. By sketching the secant lines we can determine that the average rate of change between \( x = 0 \) and \( x = 3 \) is greater than the average rate of change between \( x = 3 \) and \( x = 5 \).

(b) Is the instantaneous rate of change of the cost of producing \( x \) kilograms greater at \( x = 1 \) or at \( x = 4 \)? Explain your answer graphically.

We can see from the graph that the function is increasing faster at \( x = 1 \) than at \( x = 4 \). Therefore, the instantaneous rate of change at \( x = 1 \) is greater than the instantaneous rate of change at \( x = 4 \).

(c) What are the units of these rates of change?

The units of rate of change are obtained by dividing units of cost by units of product: thousands of dollars/kilogram.

2. For the function shown in Figure below, at what labeled points is the slope of the graph positive? Negative? At which labeled point does the graph have the greatest (i.e., most positive) slope? The least slope (i.e., negative and with the largest magnitude)?

The slope is positive at A and D; negative at C and F. The slope is most positive at A; most negative at F.